
EVALUATOR 3.0: An Efficient On-the-Fly Model-Checker for Regular Alternation-Free Mu-Calculus

Radu Mateescu

INRIA Rhône-Alpes / VASY

<http://www.inrialpes.fr/vasy/people/Radu.Mateescu>



Outline

- Introduction
- Regular alternation-free mu-calculus
- On-the-fly model-checking
- Diagnostic generation
- Implementation and applications
- Conclusion



Context of the work

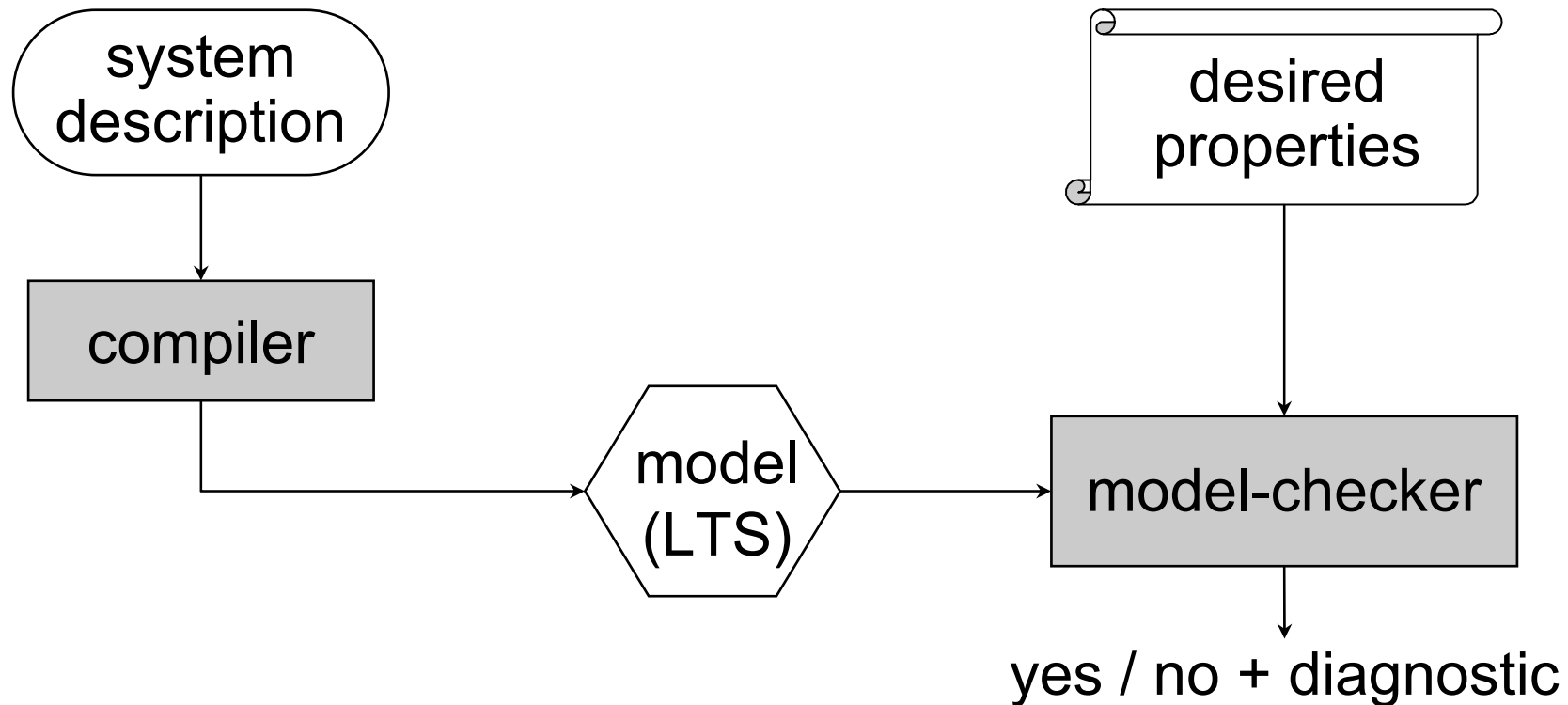
- **CADP (Caesar/Aldebaran Development Package):**
a toolbox for analysing concurrent systems
- State-of-the-art functionalities:
 - compilation, interactive simulation, verification
 - rapid prototyping, random execution, test generation
- Distributed over 280 sites (industry and academics)
- Applications:
 - 65 published case-studies
 - 13 additional research tools

<http://www.inrialpes.fr/vasy/cadp>



Model-Checking

Goal: verify that a concurrent finite-state system meets a set of desired correctness properties.



Requirements for model-checking

Expressiveness of the temporal logic

- useful temporal properties (*safety, liveness, fairness*)
- modal μ -calculus [Kozen-83] = « temporal logic assembler »

Complexity of the model-checking problem

- full μ -calculus = exponential-time
- *alternation-free* μ -calculus = linear-time

User-friendliness of the model-checker interface

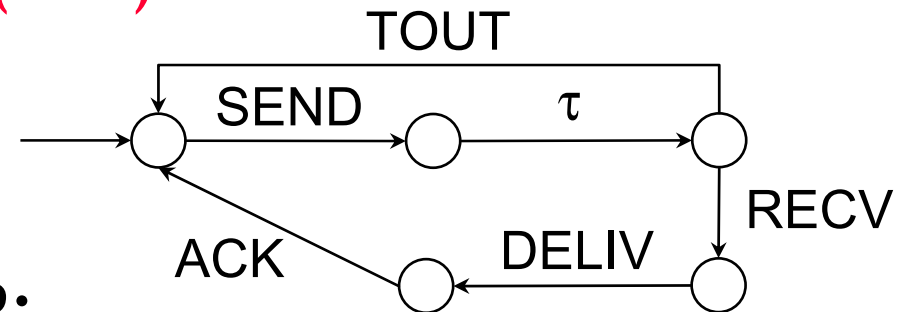
- abstraction mechanisms for defining new operators
- diagnostic generation facilities



Interpretation models

Labelled Transition System (LTS)

$$M = (S, A, T, s_0)$$



LTS representations in CADP:

- *explicit* (« predecessor » function)
 - iterative computations using sets of states
 - BCG (*Binary Coded Graphs*) environment [Garavel-92]
- *implicit* (« successor » function)
 - on-the-fly exploration of the transition relation
 - Open / Caesar environment [Garavel-98]

Regular alternation-free μ -calculus

Let $M = (S, A, T, s_0)$ be an LTS.

Action formulas (\approx ACTL):

$$\alpha ::= a \mid \neg\alpha \mid \alpha_1 \vee \alpha_2$$

Regular formulas (\approx PDL):

$$\beta ::= \alpha \mid \beta_1 \cdot \beta_2 \mid \beta_1 \mid \beta_2 \mid \beta^*$$

State formulas (\approx μ -calculus):

$$\varphi ::= F \mid \neg\varphi \mid \varphi_1 \vee \varphi_2 \mid \langle \beta \rangle \varphi \mid Y \mid \mu Y . \varphi$$

Action formulas

Let $M = (S, A, T, s_0)$. Semantics $[[\alpha]] \subseteq A$:

- $[[a]] = \{a\}$
- $[[\neg\alpha]] = A \setminus [[\alpha]]$
- $[[\alpha_1 \vee \alpha_2]] = [[\alpha_1]] \cup [[\alpha_2]]$

Derived operators:

- $T = a \vee \neg a$
- $F = \neg T$
- $\alpha_1 \wedge \alpha_2 = \neg(\neg\alpha_1 \vee \neg\alpha_2)$
- $\alpha_1 \Rightarrow \alpha_2 = \neg\alpha_1 \vee \alpha_2$
- $\alpha_1 \Leftrightarrow \alpha_2 = (\alpha_1 \Rightarrow \alpha_2) \wedge (\alpha_2 \Rightarrow \alpha_1)$



Regular formulas

Let $M = (S, A, T, s_0)$. Semantics $[[\beta]] \subseteq S \times S$:

- $[[\alpha]] = \{ (s, s') \mid \exists a \in A . (s, a, s') \in T \}$
- $[[\beta_1 \cdot \beta_2]] = [[\beta_1]] \circ [[\beta_2]]$ (composition)
- $[[\beta_1 \mid \beta_2]] = [[\beta_1]] \cup [[\beta_2]]$ (union)
- $[[\beta^*]] = [[\beta]]^*$ (star closure)

Derived operators:

- $\text{nil} = F^*$
- $\beta^+ = \beta \cdot \beta^*$



State formulas

Let $M = (S, A, T, s_0)$ and $\rho : Y \rightarrow 2^S$ a context mapping variables to state sets. Semantics $[[\varphi]] \rho \subseteq S$:

- $[[F]] \rho = \emptyset$
- $[[\neg \varphi]] \rho = S \setminus [[\varphi]] \rho$
- $[[\varphi_1 \vee \varphi_2]] \rho = [[\varphi_1]] \rho \cup [[\varphi_2]] \rho$
- $[[\langle \beta \rangle \varphi]] \rho = \{ s \in S \mid \exists (s, s') \in [[\beta]]. s' \in [[\varphi]] \rho \}$
- $[[Y]] \rho = \rho(Y)$
- $[[\mu Y . \varphi]] \rho = \bigcup_{k \geq 0} \Phi_\rho^k(\emptyset)$
where $\Phi_\rho : 2^S \rightarrow 2^S$, $\Phi_\rho(U) = [[\varphi]] \rho [U / Y]$

Derived operators:

- $[\beta] \varphi = \neg \langle \beta \rangle \neg \varphi$
- $\nu Y . \varphi = \neg \mu Y . \neg \varphi [\neg Y / Y]$



Satisfaction of state formulas

- Let $M = (S, A, T, s_0)$ and φ a state formula.

M satisfies φ ($M \models \varphi$) iff

$\forall s \in S . s \models \varphi$ iff

$[[\varphi]] = S$

- **Global model-checking:**

check a formula on all states

$$S = [[\varphi]]$$

- **Local (on-the-fly) model-checking:**

check a formula on the initial state

$$s_0 \in [[[T^*] \varphi]]$$



Safety properties

- **Absence of ERROR actions:**

$$[T^* . \text{ERROR}] F$$

- **Mutual exclusion between OPEN and CLOSE:**

$$[T^* . \text{OPEN}_1 . (\neg \text{CLOSE}_1)^* . \text{OPEN}_2] F$$

- **Alternation between SEND and RECV:**

$$[(\neg \text{SEND})^* . \text{RECV}] F \wedge$$

$$[T^* . \text{RECV} . (\neg \text{SEND})^* . \text{RECV}] F \wedge$$

$$[T^* . \text{SEND} . (\neg \text{RECV})^* . \text{SEND}] F$$

$$= [((\text{nil} \mid (T^* . \text{RECV})) . (\neg \text{SEND})^* . \text{RECV}) \mid \\ (T^* . \text{SEND} . (\neg \text{RECV})^* . \text{SEND})] F$$



Liveness properties

- **Deadlock freedom:**

$$[T^*] \langle T \rangle T$$

- **Potential reachability of a RECV after a SEND and some ERRORS:**

$$\langle T^* . \text{SEND} . (T^* . \text{ERROR})^* . T^* . \text{RECV} \rangle T$$

- **Inevitable reachability of a GRANT after a REQ:**

$$[T^* . \text{REQ}] \mu Y . \langle T \rangle T \wedge [\neg \text{GRANT}] Y$$



Fairness properties

- Absence of livelocks (tau-circuits) :

$$\neg vY . \langle \text{tau} \rangle Y =$$

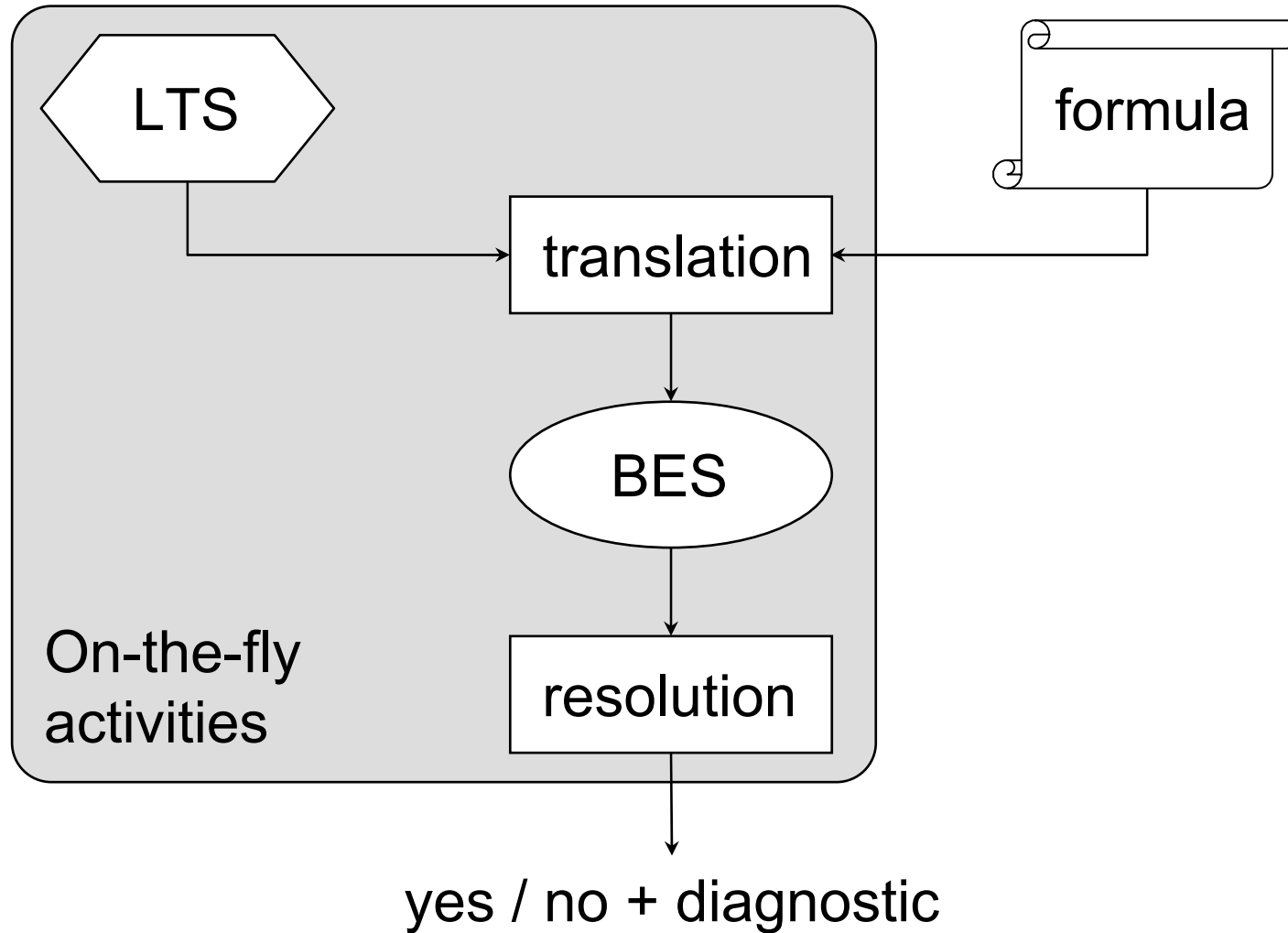
$$\mu Y . [\text{tau}] Y$$

- Fair reachability (by skipping circuits) of a RECV after a SEND:

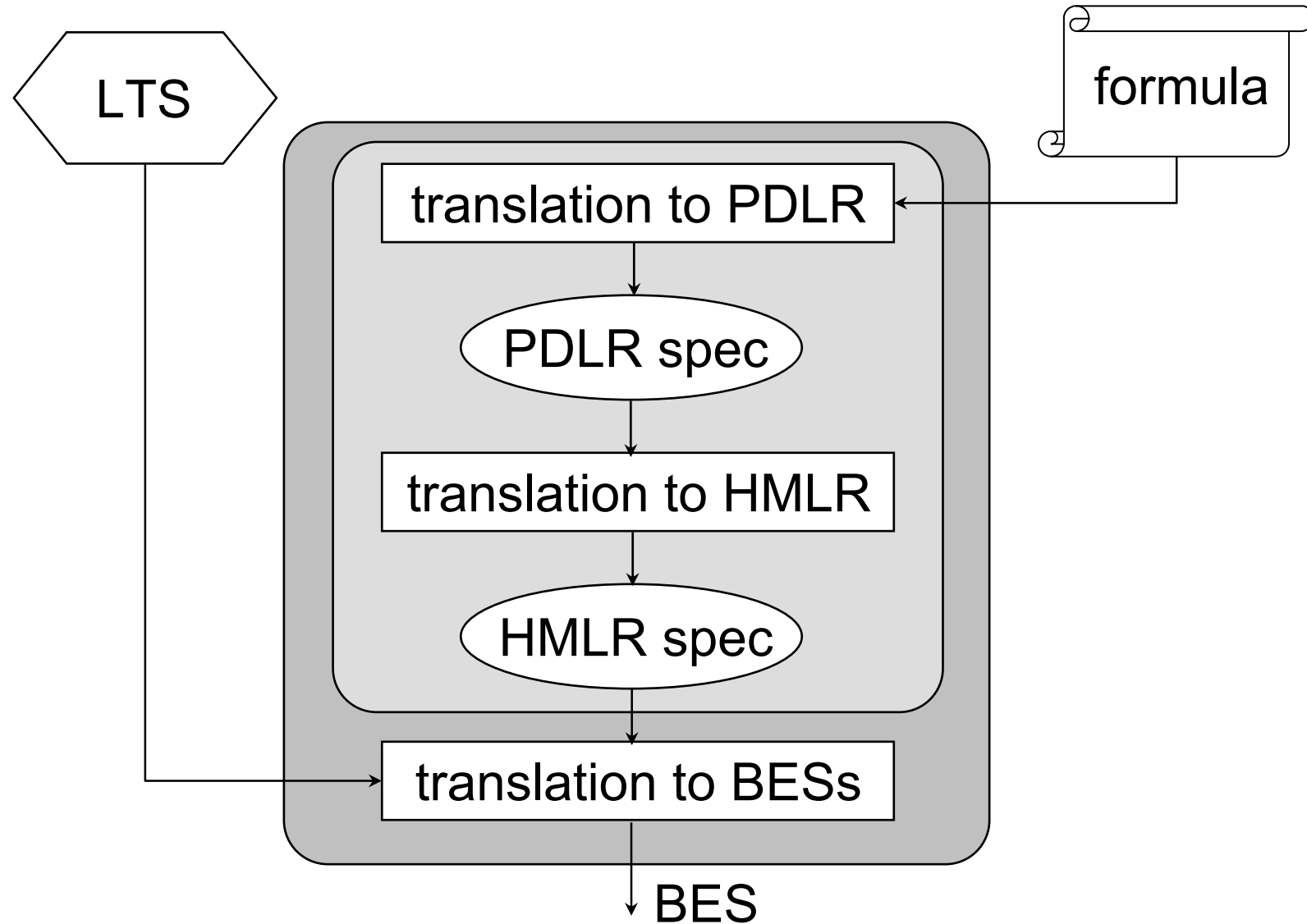
$$[T^* . \text{SEND} . (\neg \text{RECV})^*] \langle T^* . \text{RECV} \rangle T$$



On-the-fly model-checking



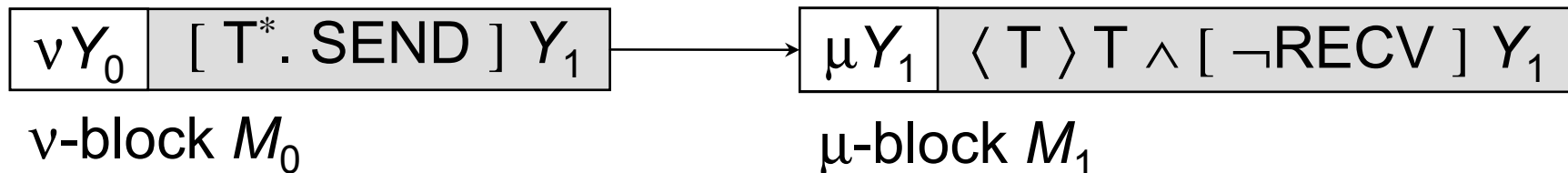
Translation to BESs



Translation to PDL with recursion

State formula (*expanded*):

$$vY_0 . [T^* . \text{SEND}] \mu Y_1 . \langle T \rangle T \wedge [\neg \text{RECV}] Y_1$$



PDLR specification:

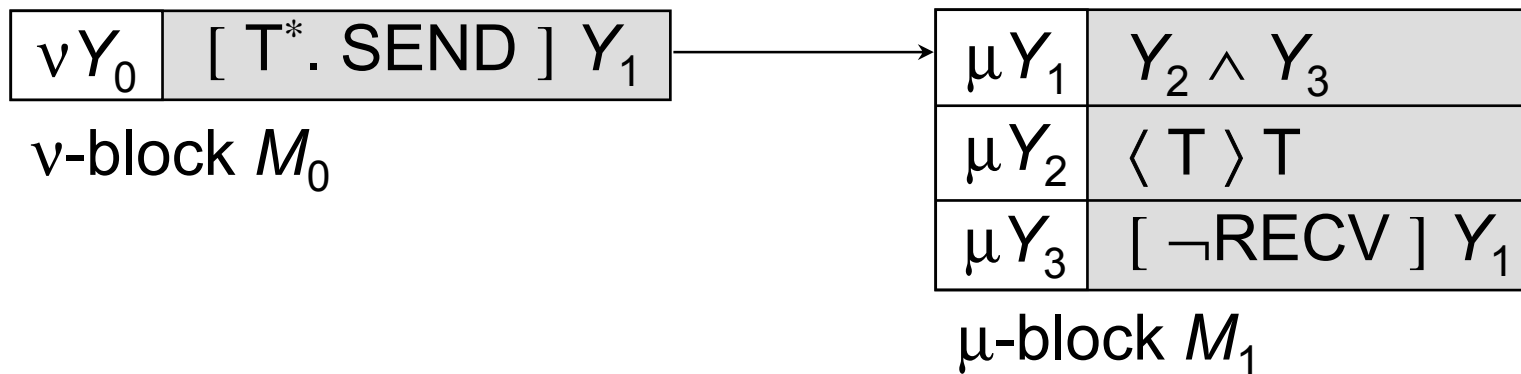
$$(Y_0, \{ Y_0 =_v [T^* . \text{SEND}] Y_1 \} \\ \{ Y_1 =_\mu \langle T \rangle T \wedge [\neg \text{RECV}] Y_1 \})$$



Simplification

PDLR specification:

$$(Y_0, \{ Y_0 =_v [T^* . \text{SEND}] Y_1 \} \\ \{ Y_1 =_\mu \langle T \rangle T \wedge [\neg \text{RECV}] Y_1 \})$$



Simple PDLR specification:

$$(Y_0, \{ Y_0 =_v [T^* . \text{SEND}] Y_1 \} \\ \{ Y_1 =_\mu Y_2 \wedge Y_3, Y_2 =_\mu \langle T \rangle T, Y_3 =_\mu [\neg \text{RECV}] Y_1 \})$$

Translation to HML with recursion

Simple PDLR specification:

$$(Y_0, \{ Y_0 =_v [T^* . \text{SEND}] Y_1 \} \\ \{ Y_1 =_\mu Y_2 \wedge Y_3, Y_2 =_\mu \langle T \rangle T, Y_3 =_\mu [\neg \text{RECV}] Y_1 \})$$

vY_0	$Y_4 \wedge Y_5$
vY_4	$[\text{SEND}] Y_1$
vY_5	$[T] Y_0$

v-block M_0

μY_1	$Y_2 \wedge Y_3$
μY_2	$\langle T \rangle T$
μY_3	$[\neg \text{RECV}] Y_1$

μ -block M_1

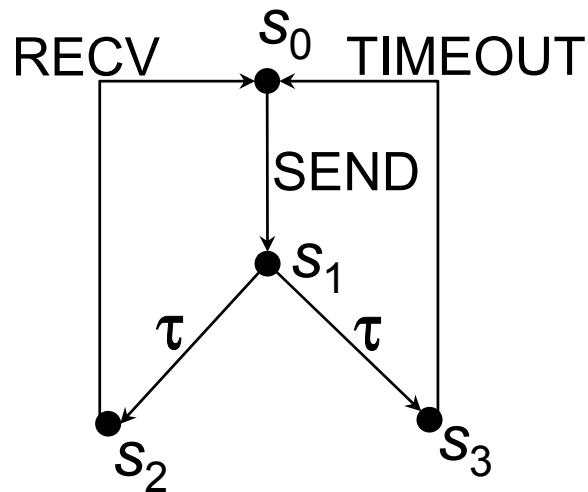
Simple HMLR specification:

$$(Y_0, \{ Y_0 =_v Y_4 \wedge Y_5, Y_4 =_v [\text{SEND}] Y_1, Y_5 =_v [T] Y_0 \} \\ \{ Y_1 =_\mu Y_2 \wedge Y_3, Y_2 =_\mu \langle T \rangle T, Y_3 =_\mu [\neg \text{RECV}] Y_1 \})$$

Translation to BESs

$\forall Y_0$	$Y_4 \wedge Y_5$
$\forall Y_4$	$[\text{SEND}] Y_1$
$\forall Y_5$	$[\text{T}] Y_0$

μY_1	$Y_2 \wedge Y_3$
μY_2	$\langle \text{T} \rangle \text{T}$
μY_3	$[\neg \text{RECV}] Y_1$



Boolean variables: $x_{i,j} \equiv s_i \models Y_j$

$X_{0,0} =_v X_{0,4} \wedge X_{0,5}$
$X_{0,4} =_v X_{1,1}$
$X_{0,5} =_v X_{1,0}$
$X_{1,0} =_v X_{1,4} \wedge X_{1,5}$
$X_{1,4} =_v \text{T}$
$X_{1,5} =_v X_{2,0} \wedge X_{3,0}$
$X_{2,0} =_v X_{2,4} \wedge X_{2,5}$
$X_{2,4} =_v \text{T}$
$X_{2,5} =_v X_{0,0}$
$X_{3,0} =_v X_{3,4} \wedge X_{3,5}$
$X_{3,4} =_v \text{T}$
$X_{3,5} =_v X_{0,0}$

$X_{1,1} =_\mu X_{1,2} \wedge X_{1,3}$
$X_{1,2} =_\mu \text{T}$
$X_{1,3} =_\mu X_{2,1} \wedge X_{3,1}$
$X_{2,1} =_\mu X_{2,2} \wedge X_{2,3}$
$X_{2,2} =_\mu \text{T}$
$X_{2,3} =_\mu \text{T}$
$X_{3,1} =_\mu X_{3,2} \wedge X_{3,3}$
$X_{3,2} =_\mu \text{T}$
$X_{3,3} =_\mu X_{0,1}$
$X_{0,1} =_\mu X_{0,2} \wedge X_{0,3}$
$X_{0,2} =_\mu \text{T}$
$X_{0,3} =_\mu X_{1,1}$



Boolean Equation Systems

- **Syntax:**

$$M = \{ x_i =_{\sigma} op_i X_i \}_{1 \leq i \leq n}$$

where $\sigma \in \{\mu, \nu\}$, $x_i \in X$, $op_i \in \{\vee, \wedge\}$, $X_i \subseteq X$ for all $i=1..n$

- **Semantics:**

Bool = {F, T} and $\delta : X \rightarrow \mathbf{Bool}$

$$[[op \{x_1, \dots, x_k\}]] \delta = \delta(x_1) op \dots op \delta(x_k)$$

$$[[M]] \delta = \sigma \Psi_{\delta}$$

where $\Psi_{\delta} : \mathbf{Bool}^n \rightarrow \mathbf{Bool}^n$,

$$\Psi_{\delta}(b_1, \dots, b_n) = ([[op_i X_i]] \delta [b_1 / x_1, \dots, b_n / x_n])_{1 \leq i \leq n}$$



Extended Boolean Graphs

- Slight extension of *boolean graphs* [Andersen-94]
= alternative graphical representation of BESs
- To any BES $M = \{ x_i =_{\sigma} op_i X_i \}_{1 \leq i \leq n}$ corresponds an *extended boolean graph* (EBG) $G = (V, E, L, F)$:

$$V = \{ x_1, \dots, x_n \}$$

vertex set

$$E = \{ x_i \rightarrow x_j \mid x_j \in X_i \}$$

edge set

$$L : V \rightarrow \{ \vee, \wedge \}, L(x_i) = op_i$$

vertex labeling

$$F \subseteq V$$

frontier

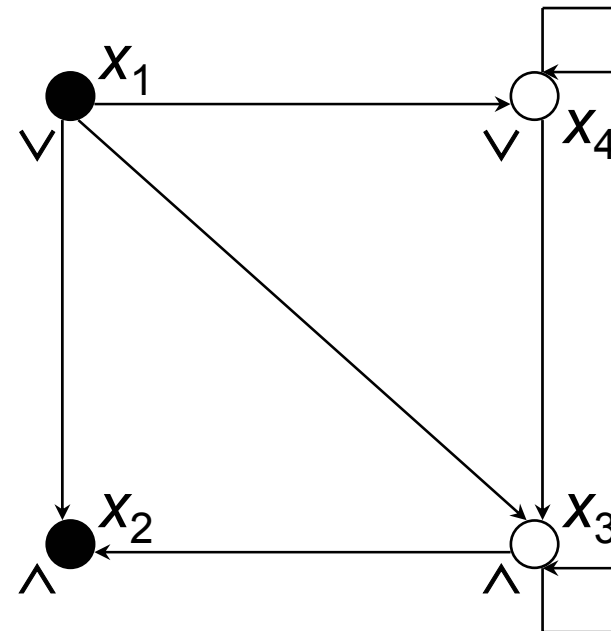


Example

BES

$$\left\{ \begin{array}{l} X_1 =_{\mu} X_2 \vee X_3 \vee X_4 \\ X_2 =_{\mu} \top \\ X_3 =_{\mu} X_2 \wedge X_3 \\ X_4 =_{\mu} X_3 \vee X_4 \end{array} \right.$$

EBG



Characterization of BES solution

Let $M = \{ x_i =_{\mu} op_i X_i \}_{1 \leq i \leq n}$ with $G = (V, E, L, F)$.

Let P_{\vee} and P_{\wedge} be two propositions compatible with L .

Consider two μ -calculus formulas:

$$\mathbf{EX} = \mu Y . (P_{\vee} \wedge \langle - \rangle Y) \vee (P_{\wedge} \wedge [-] Y)$$

$$\mathbf{CX} = \nu Y . (P_{\vee} \wedge [-] Y) \vee (P_{\wedge} \wedge \langle - \rangle Y)$$

called **example formula** and **counterexample formula**.

Theorem 1:

$$[[M]]_i = T \iff x_i \models_G \mathbf{EX}$$

for all $1 \leq i \leq n$

Example

$$EX = \mu Y . (P_{\vee} \wedge \langle - \rangle Y) \vee (P_{\wedge} \wedge [-] Y)$$

$$CX = \nu Y . (P_{\vee} \wedge [-] Y) \vee (P_{\wedge} \wedge \langle - \rangle Y)$$

$$EX^0 = \emptyset$$

$$EX^1 = \{ x_2 \}$$

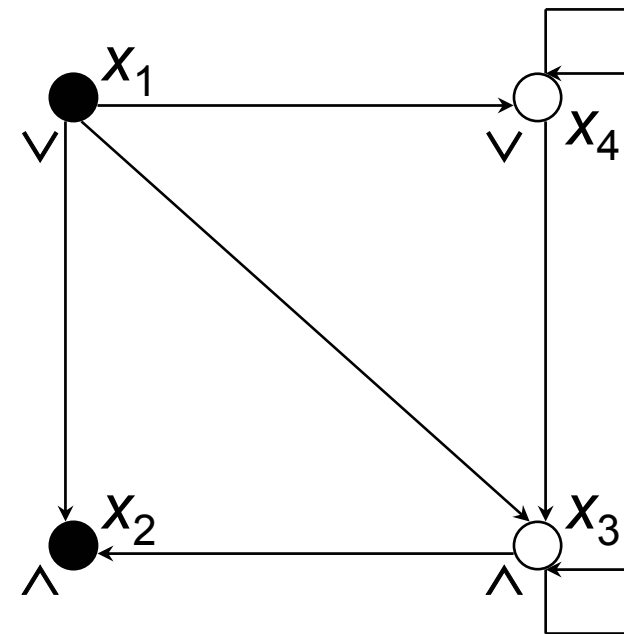
$$EX^2 = \{ x_1, x_2 \} = [[EX]]$$

$$CX^0 = \{ x_1, x_2, x_3, x_4 \}$$

$$CX^1 = \{ x_1, x_3, x_4 \}$$

$$CX^2 = \{ x_3, x_4 \} = [[CX]]$$

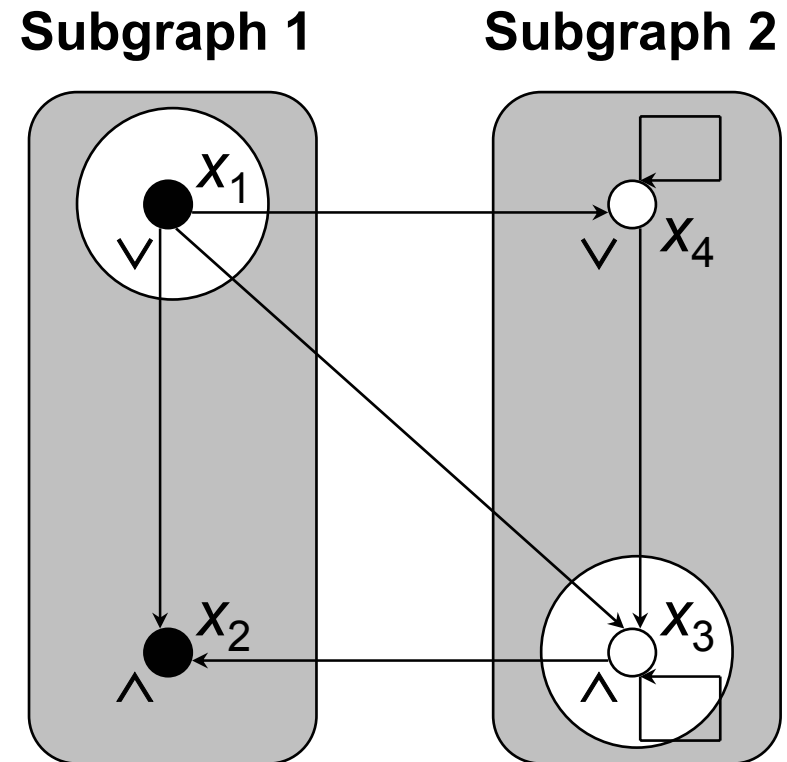
$$\begin{cases} x_1 =_{\mu} x_2 \vee x_3 \vee x_4 \\ x_2 =_{\mu} \top \\ x_3 =_{\mu} x_2 \wedge x_3 \\ x_4 =_{\mu} x_3 \vee x_4 \end{cases}$$



Subgraphs and frontiers

$G_1 = (V_1, E_1, L_1, F_1)$ subgraph of $G_2 = (V_2, E_2, L_2, F_2)$
(noted $G_1 \leq G_2$) iff:

- $V_1 \subseteq V_2$
- $E_1 \subseteq E_2$
- $(E_2 \setminus E_1) \upharpoonright_{V_1} = (E_2 \setminus E_1) \upharpoonright_{F_1}$
- $F_2 \cap V_1 \subseteq F_1$
- $L_1 = L_2 \upharpoonright_{V_1}$



Solution-closed EBGs

An EBG $G_1 = (V_1, E_1, L_1, F_1)$ is *solution-closed* iff:

$$G_1 \leq G_2 \Rightarrow [[\mathbf{EX}]]_{G_1} = [[\mathbf{EX}]]_{G_2} \cap V_1$$

for any $G_2 = (V_2, E_2, L_2, F_2)$.

Theorem 2:

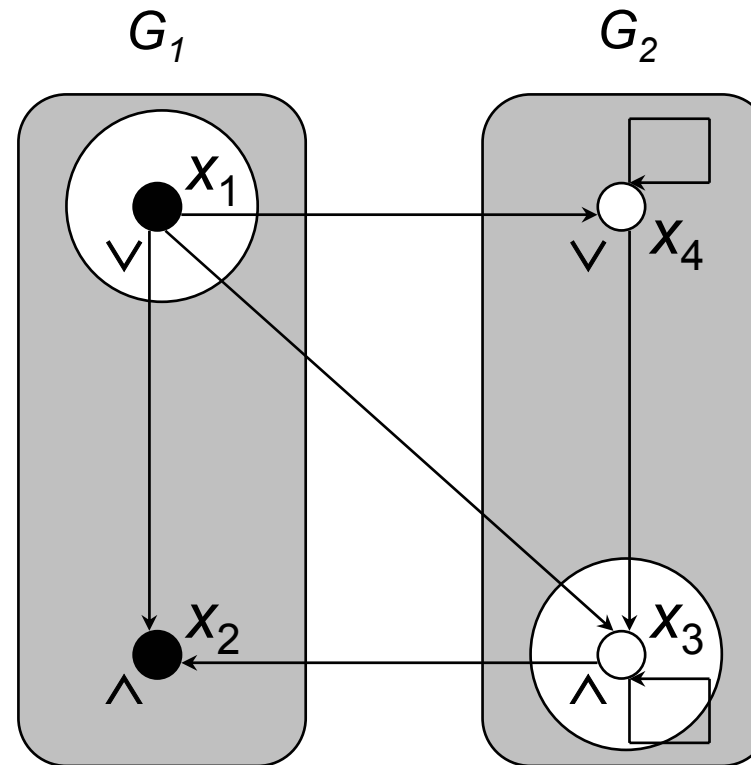
$G = (V, E, L, F)$ is solution-closed iff:

$$F \subseteq [[(P_{\vee} \wedge \mathbf{EX}) \vee (P_{\wedge} \wedge \mathbf{CX})]]_G$$

Example

$$F_1 = \{x_1\} \in [[\mathbf{EX}]]_{G_1}$$

$$F_2 = \{x_3\} \in [[\mathbf{CX}]]_{G_2}$$

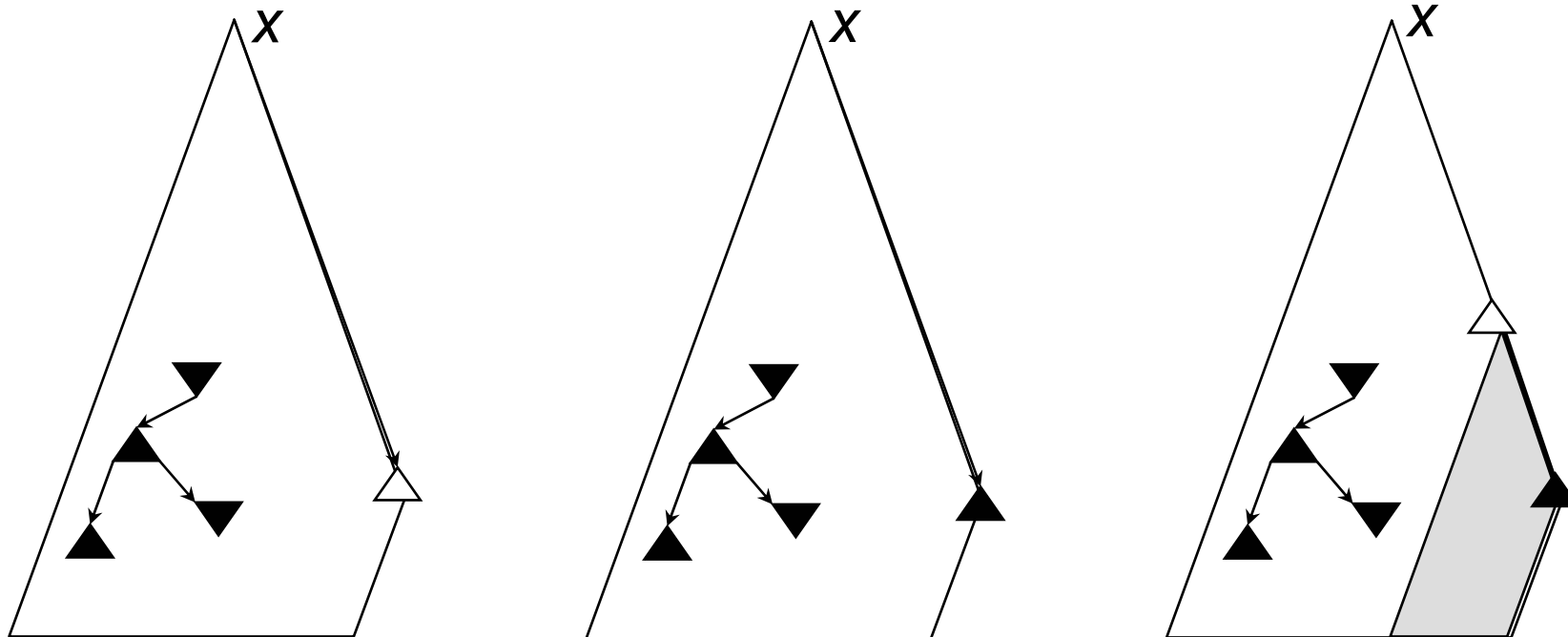


Two solution-closed subgraphs

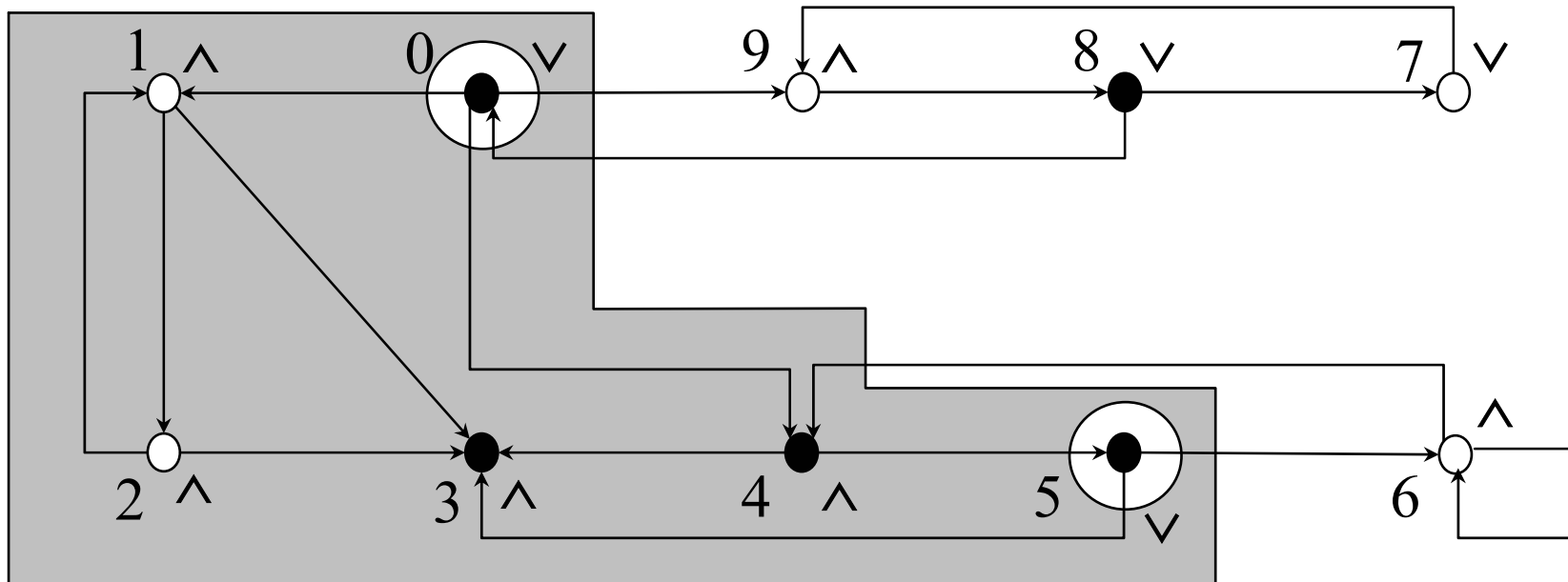
Local resolution algorithm

Idea: compute a solution-closed subgraph containing the boolean variable of interest

SOLVE = DFS of the boolean graph with back-propagation of **EX**-vertices [Andersen-94, Mateescu-Sighireanu-00]



Example



A solution-closed subgraph rooted at x_0
computed by SOLVE

Diagnostics

Let $G = (V, E, L, F)$ be an EBG and $x \in V$.

A *diagnostic* for $x =$
a subgraph $G' \leq G$ such that

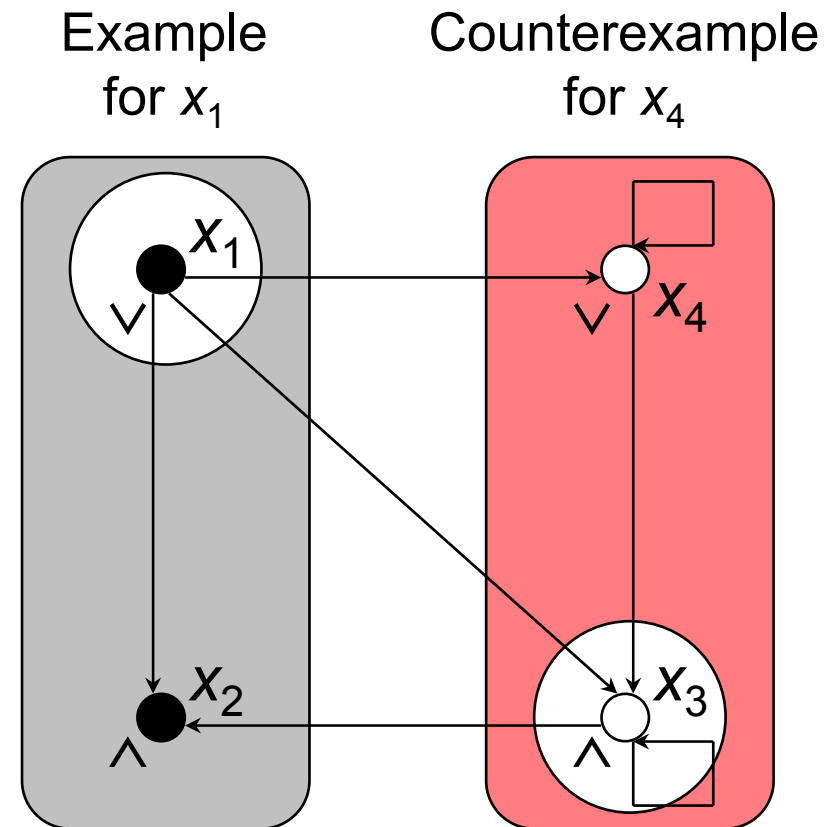
$$x \models_G \mathbf{EX} \Leftrightarrow x \models_{G'} \mathbf{EX}$$

If $x \models_G \mathbf{EX}$ then

$G' =$ *example* for x

If $x \models_G \mathbf{CX}$ then

$G' =$ *counterexample* for x



Minimal diagnostic characterization

$G = (V, E, L, F)$ *diagnostic* for $x \in V$ iff G *solution-closed*.

Theorem 3:

$G = (V, E, L, F)$ is a minimal example for $x \in V$ (wrt. \leq) iff:

- a) G is an **EX**-model
- b) $\forall y \in V . L(y) = \vee \Rightarrow |E(y)| = 1$
- c) $V = E^*(x)$
- d) $F = \{ y \in V \mid L(y) = \vee \}$

Precomputation step (only for minimal examples)

Start with a solution-closed subgraph computed by SOLVE.

Define the increasing chain

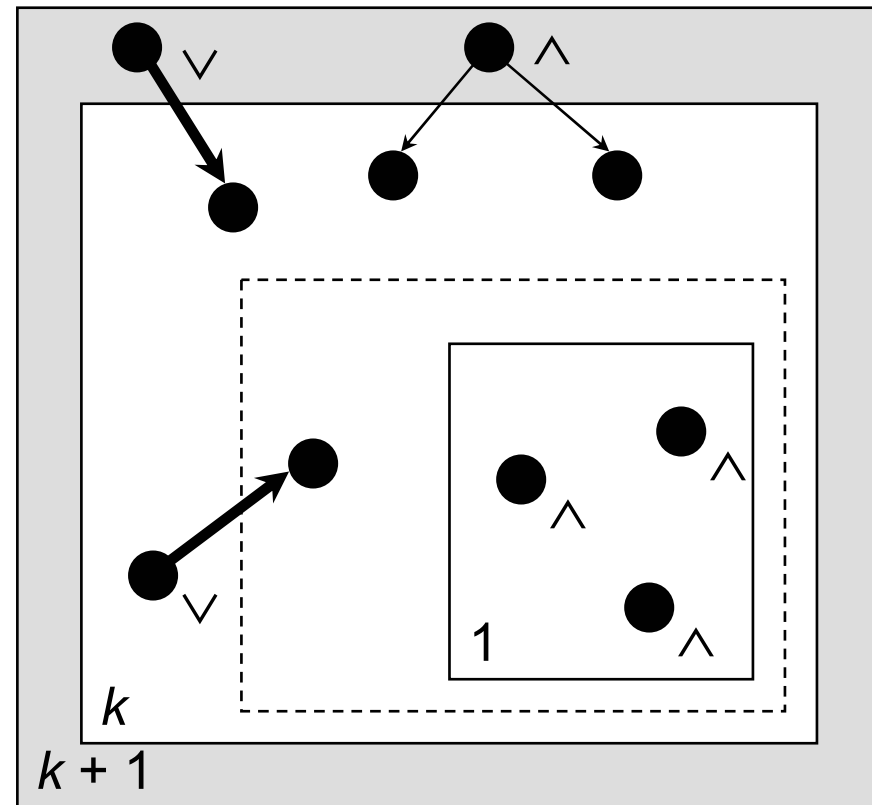
- $EX^0 = \emptyset$
- $EX^{k+1} = \Phi^{EX}(EX^k)$

where

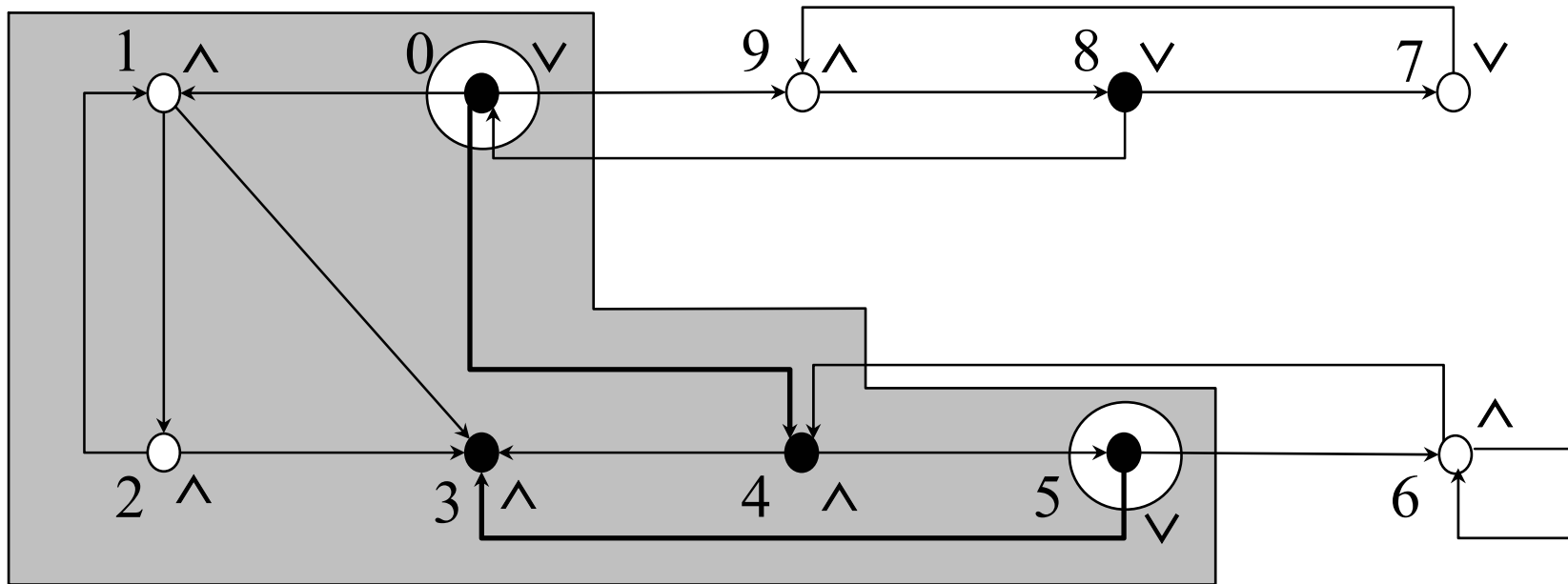
$$\Phi^{EX}(U) = [[(P_{\vee} \wedge \langle - \rangle Y) \vee (P_{\wedge} \wedge [-] Y)]] [U / Y]$$

Iterate EX^k until $EX^{k+1} = EX^k$

For each \vee -vertex in EX^{k+1}
store a successor in EX^k



Example



Precomputation of the « good » successors
for v-vertices in a solution-closed subgraph

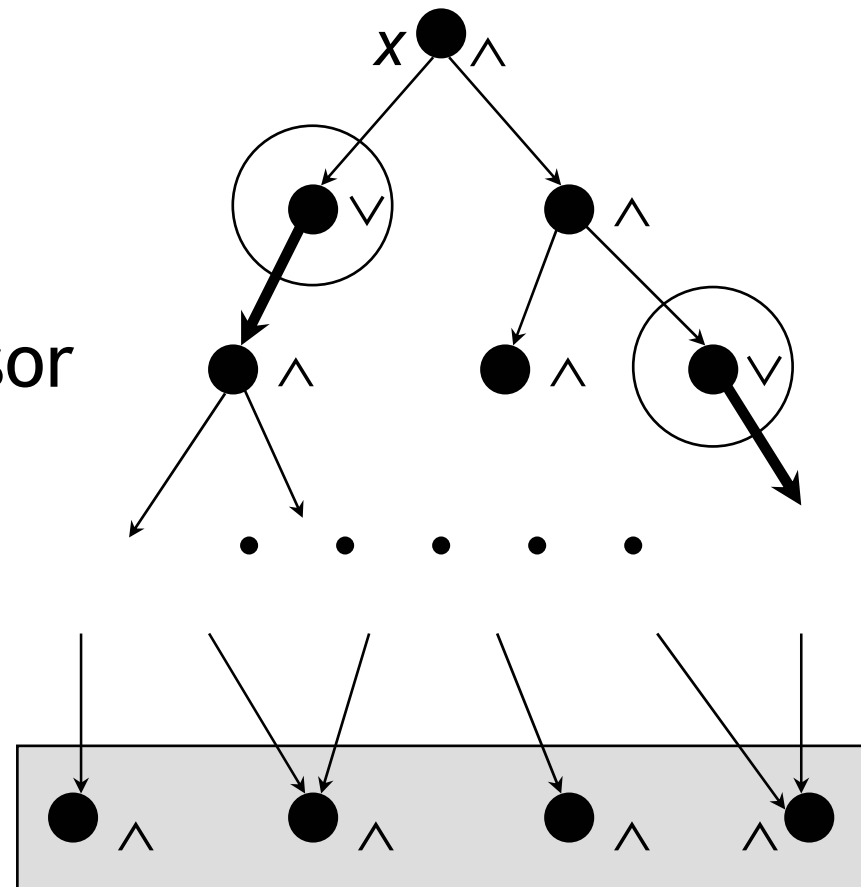
Generation of minimal examples

Forward exploration of $[[\text{EX}]]$ _G starting at x

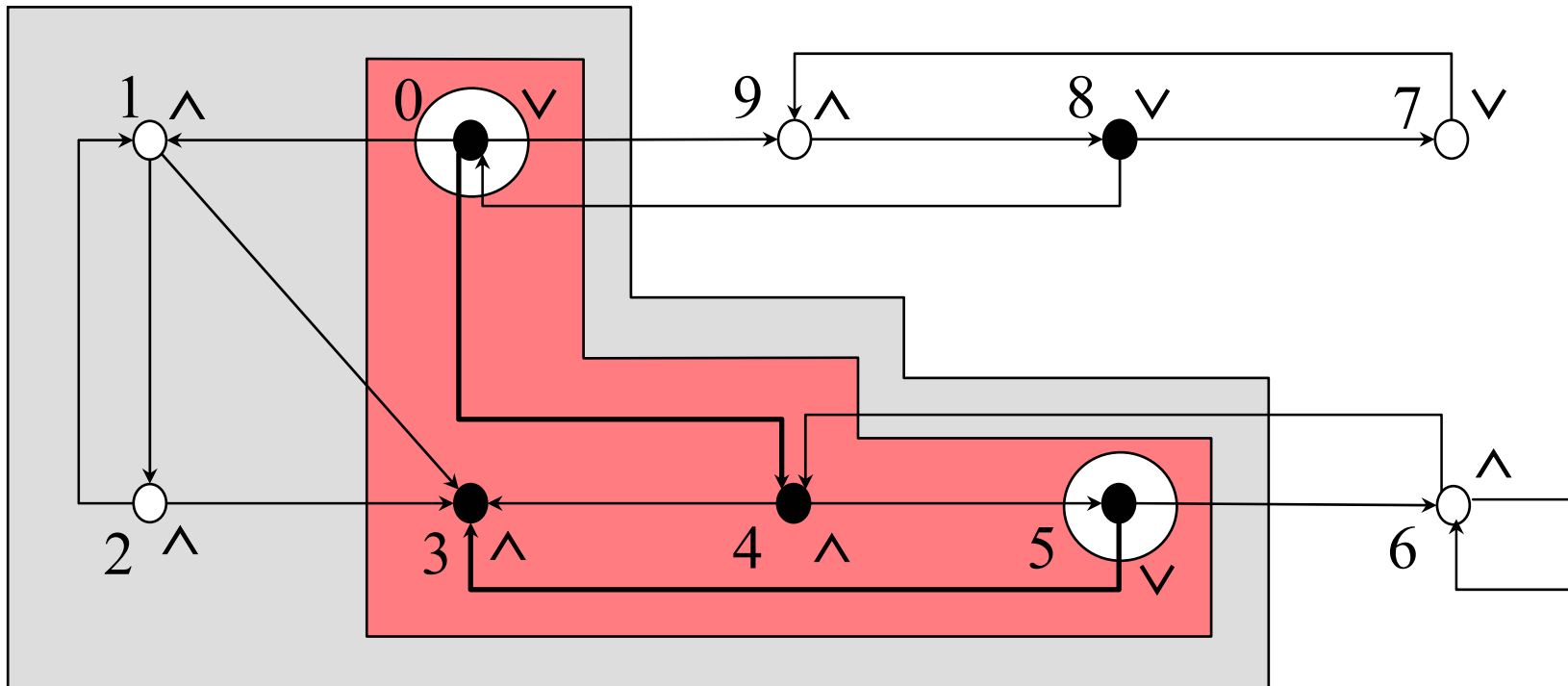
for each y reached do

- if $L(y) = \vee$ then
follow the “good” successor
- if $L(y) = \wedge$ then
follow all successors

until reach \wedge -sink vertices



Example



A minimal example for x_0

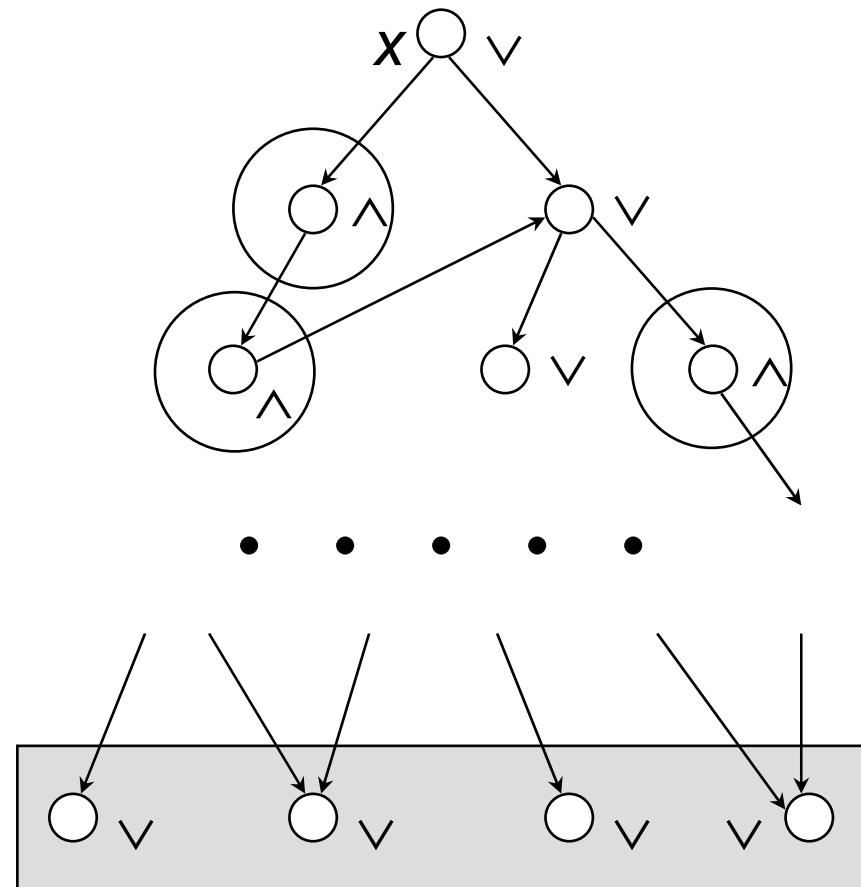
Generation of minimal counterexamples

Forward exploration of $[[CX]]_G$ starting at x

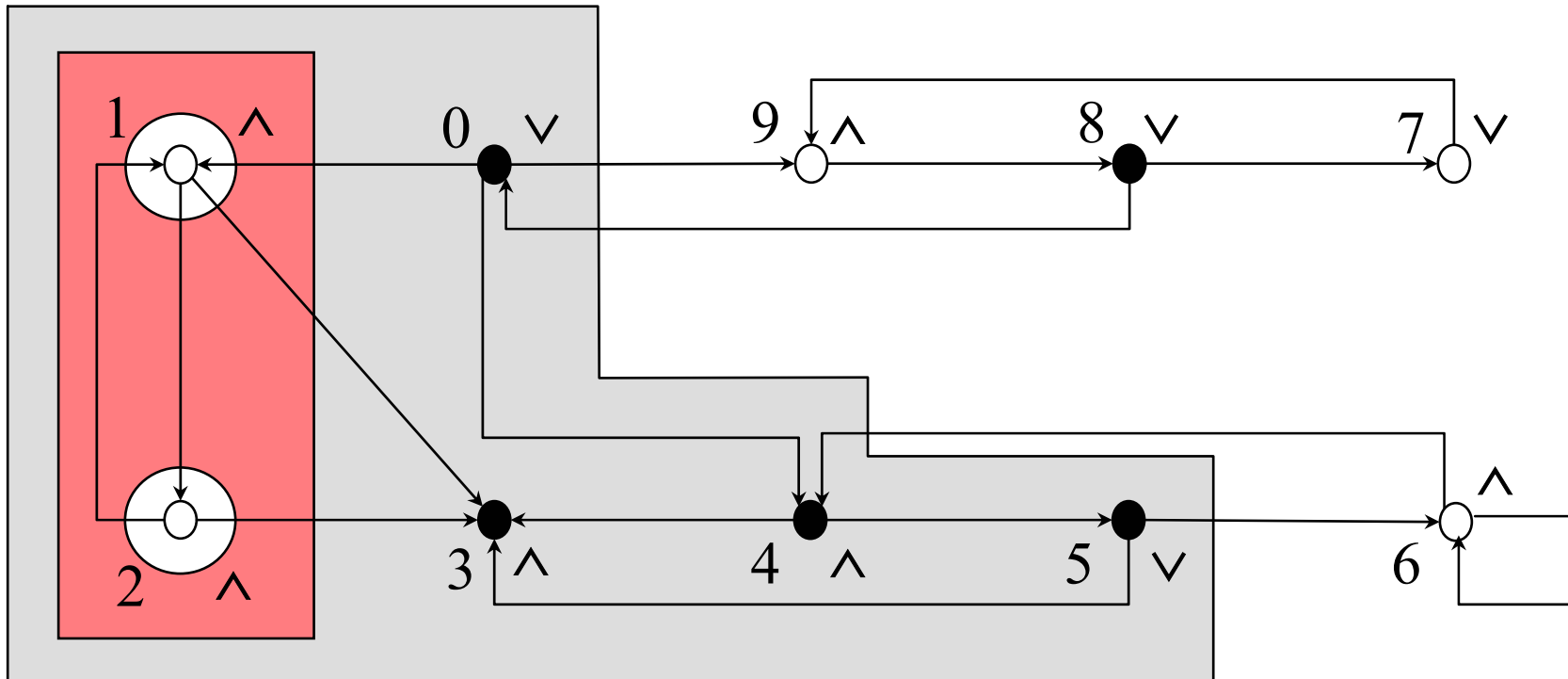
for each y reached do

- if $L(y) = \wedge$ then
follow a single successor
- if $L(y) = \vee$ then
follow all successors

until reach \vee -sink vertices
or loop back



Example

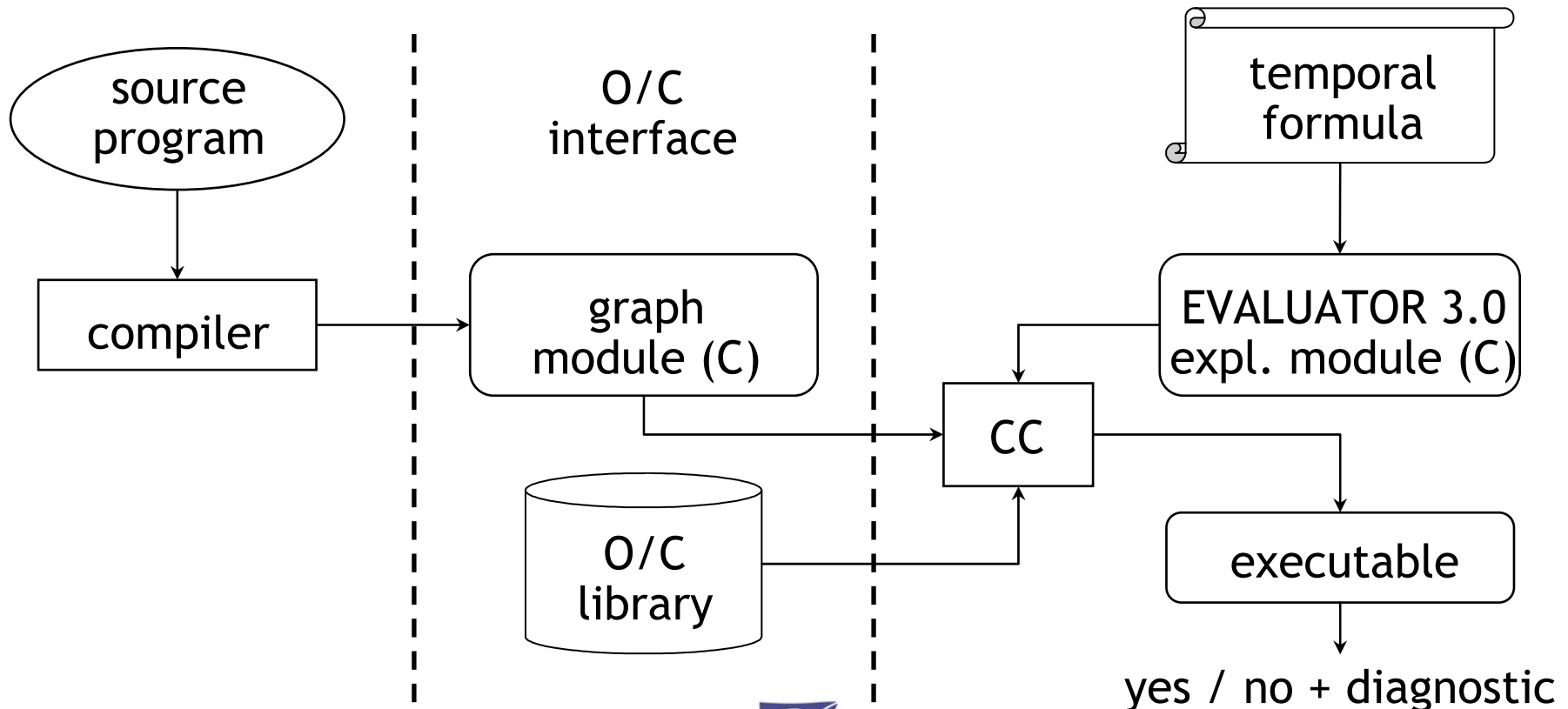


A minimal counterexample for x_1

Implementation

Evaluator 3.0 on-the-fly model-checker:

- Developed within CADP using the **Open/Caesar** generic environment [Garavel-98] for on-the-fly exploration of LTSs
- About 10,000 lines of code (SYNTAX + FNC-2 + C)



Additional operators

Macro-definition (overloaded) and library inclusion

- Libraries encoding the operators of **CTL** and **ACTL**

$$EU(\varphi_1, \varphi_2) = \mu Y . \varphi_2 \vee (\varphi_1 \wedge \langle T \rangle Y)$$

$$EU(\varphi_1, \alpha_1, \alpha_2, \varphi_2) = \mu Y . \langle \alpha_2 \rangle \varphi_2 \vee (\varphi_1 \wedge \langle \alpha_1 \rangle Y)$$

- Libraries of high-level property patterns [Dwyer-99]

- Property classes:

- Absence, existence, universality, precedence, response

- Property scopes:

- Globally, before a , after a , between a and b , after a until b

- More info:

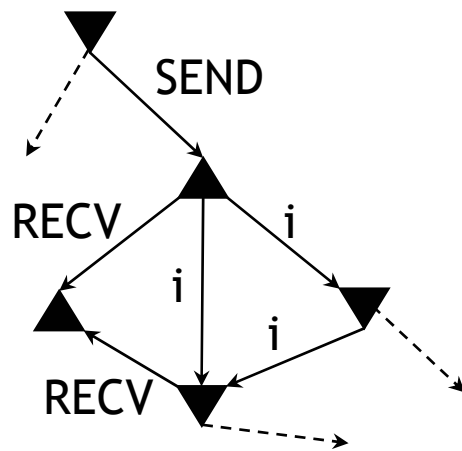
- <http://www.inrialpes.fr/vasy/cadp/resources>

- Library of robot-task specific properties (ORCCAD)

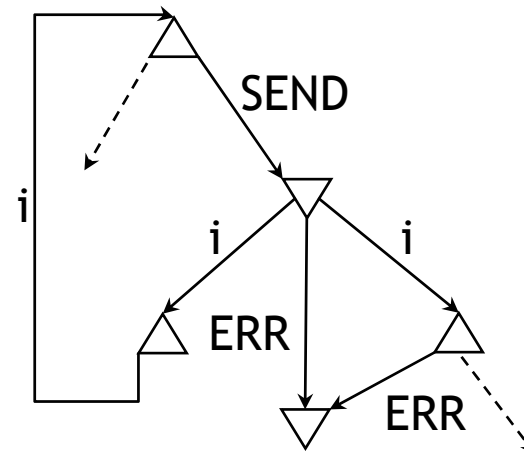


Diagnostic features

- Interpret EBG-based diagnostics in terms of the LTS
 - Keep only the edges related to LTS transitions



Example



Counterexample

- Full diagnostics (examples and counterexamples)
 - Facilitate the understanding of temporal logic formulas
 - Useful for debugging and teaching purposes

Guided simulation

Evaluator 3.0 used with OCIS for guided simulation:

- Generate a sequence matching a regular formula β (e.g., as a diagnostic for a regular modality)
 - example for $\langle \beta \rangle T$
 - counterexample for $[\beta] F$
- Load the sequence in the OCIS simulator of CADP
- Continue the simulation step-by-step

Allows to inspect regions of the LTS where problems are suspected (e.g., feature interaction detection)

Case studies

(<http://www.inrialpes.fr/vasy/cadp/case-studies>)

- **SPLICE coordination architecture** (CWI + Thalès Nederland)
- **GPRS mobile data packet radio service** (U. Ottawa)
- **Air traffic control system** (U. Glasgow)
- **Steam-boiler system** (OBLOG)
- **Truck lifting system** (CWI + Add-Controls)
- **Distributed locker system** (ERICSSON)
- **Dynamic reconfiguration protocol** (INRIA + Bull)
- **Embedded system on Lynx helicopters** (CWI + Royal Navy)
- **Javaspaces architecture** (CWI + Sun Microsystems)
- **Needham-Schroeder authentication protocol** (CWI)
- **Video-on-demand multimedia system** (LFCIA + ERICSSON)



Conclusion

Already done:

- Succinct translation of regular alt-free μ -calculus in BESs
- Efficient on-the-fly BES resolution algorithm
- Generation of examples and counterexamples
- Evaluator 3.0 implemented in CADP using Open/Caesar
- 11 published case-studies
- Rhône-Alpes IT Award (November 2002)

Future work:

- Extension with data (Evaluator 4.0):
[SEND ?m:Msg] < T*. RECV !m > T



References

- **Diagnostic generation:**

- R. Mateescu, Efficient Diagnostic Generation for Boolean Equation Systems, *Proc. of TACAS'00 (Berlin, Germany)*, LNCS vol. 1785, pp. 251-265, Springer Verlag, March 2000. Full version available as INRIA Research Report RR-3861.

- **EVALUATOR 3.0:**

- R. Mateescu and M. Sighireanu. Efficient On-the-Fly Model-Checking for Regular Alternation-Free Mu-Calculus, *Science of Computer Programming* 46(3):255-281, March 2003. Short version available as INRIA Research Report RR-3899.
- Rhône-Alpes IT Award (Lyon, November 2002):
http://www.inrialpes.fr/vasy/Press/award_2002.html

