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# State Space Reduction for Process Algebra Specifications

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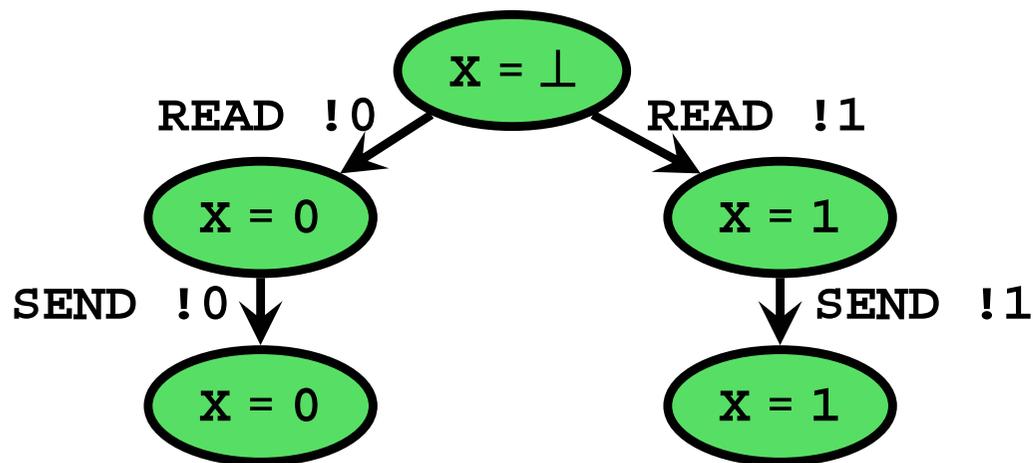
# Context

- **CADP**: widespread verification toolbox
  - 307 licenses, 74 case studies, 17 tools using CADP
  - <http://www.inrialpes.fr/vasy/cadp>
- **LOTOS**: international standard (ISO 8807)
  - Based on algebraic methodology
  - Abstract data types and process algebra
- **LOTOS-Compilers of CADP**
  - CAESAR.ADT (data types), CAESAR (processes)
  - Generation of labeled transition systems (graphs)
  - Used in 32 demos and 60 case-studies

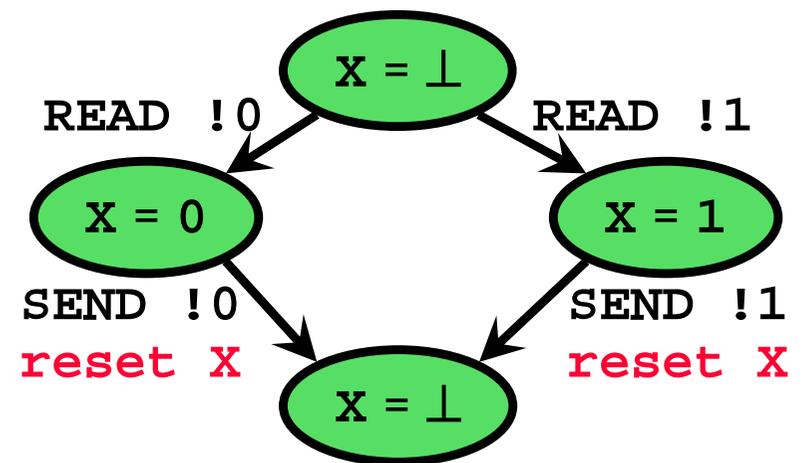


# Enumerative Verification

- Classical problem: **state explosion**
- Several techniques - here **resetting variables**
- Graf-Richier-Rodríguez-Voiron 1989:  
Manual insertion of resets in an imperative language
- **Example:** “**READ ?X:bit; SEND !X; stop**”



without reset



with reset



# Resetting Variables (1 / 3)

- Manual insertion of resets

Error-prone and **impossible** in “assign-once” languages

- Garavel 1992

- Translate LOTOS to structured Petri nets with variables



- “Syntactic criterion”:

reset variables if places of a process loose their token

- Significant state space reduction (CAESAR 4.2)



# Resetting Variables (2/3)

- Galvez-Garavel 1993 (MSc thesis, Grenoble)
  - Attempt of a more precise analysis
  - Local and global data-flow analysis
  - Automatic insertion of resets
  - Successful state space reduction

But: errors in a small number of examples  
Strong bisimulation is not preserved!

Reason not understood  $\Rightarrow$  not embedded in CAESAR

- This paper
  - Understanding of the errors
  - Solution



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# Resetting Variables (3/3)

## Related work

- **Dong-Ramakrishnan 1999**
  - Same syntactic criterion as CAESAR
  - Removing variables instead of resetting variables
- **Holzmann 1999**
  - Imperative language
  - Simpler model: *flat* collection of processes
- **Bozga-Fernandez-Ghirvu 1999**
  - Simpler model: *flat* collection of processes
  - But: Correctness proofs



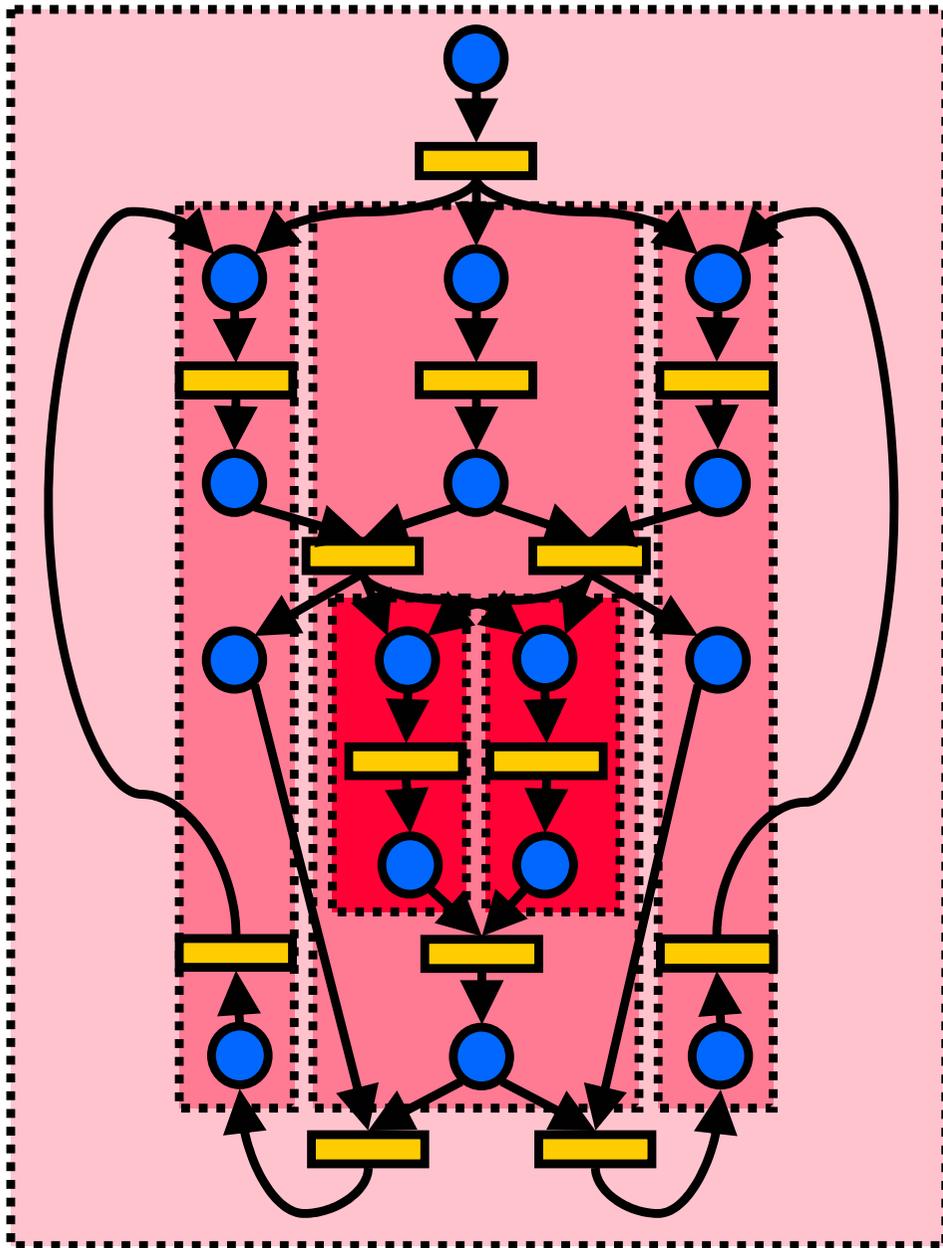
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# Network Model of CAESAR

(section 2 of the paper)



# Network Model of CAESAR (1/2)



## Structured Petri Nets

- Places 
- Transitions 
- Units 
  - Partition of the places
  - Subunit relation:  $\subseteq$

## Properties of units

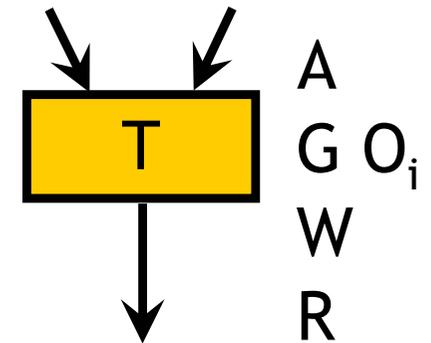
- Tree shaped hierarchy
- At most 1 marked place
- $U_1$  and  $U_2 \subseteq U_1$  are not marked simultaneously



# Network Model of CAESAR (2/2)

## Typed variables

- Attached to units
- Modified by transitions:  
Action A, offer O, guard W, reaction R



## Properties of variables

- Variables are defined before used
- Shared variables are read-only

In the LOTOS behavior: “G ?X:S; (P1 ||| P2)”

- “X” can be read by “P1” and “P2”
- “X” cannot be modified by “P1” or “P2”



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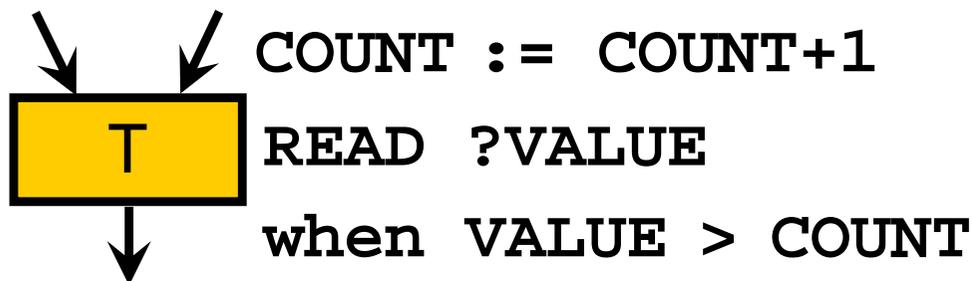
# Local Data-Flow Analysis

(section 3 of the paper)



# Local Data-Flow Analysis

- Intra-transition
- Predicates on transition  $T$  and variable  $X$   
defined by structural induction on  $T$  (i.e., A, O, W, R)
  - **$use(T, X)$** : value of  $X$  accessed by  $T$
  - **$def(T, X)$** : value of  $X$  defined *at the end* of  $T$
  - **$use\_before\_def(T, X)$** : value of  $X$  accessed *at the beginning* of  $T$ , i.e., *before* a possible *redefinition*
- Example



$def(T, COUNT), def(T, VALUE)$   
 $use(T, COUNT), use(T, VALUE)$   
 $use\_before\_def(T, COUNT)$



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# Global Data-Flow Analysis

(section 4 of the paper)



# Global Data-Flow Analysis

- **Inter-transition**: combine local results
- Classically (sequential programs)  
compute fixed point on (control-flow) graph
- **Principal difference**: **Concurrency**  
Petri nets instead of graphs
- **Idea**: **abstract Petri nets to graphs**
  - Nodes: transitions
  - Arcs: successor relation " $T_1 \rightarrow T_2$ "



# Abstracting Networks to Graphs

Several possibilities:

- **Good precision: based on reachable markings**
  - “ $T_1 \rightarrow_M T_2$ ” iff exists firable sequence “...,  $T_1, T_2$ ”
  - State explosion possible
- **Poor precision: connection by places**
  - “ $T_1 \rightarrow T_2$ ” iff  $(\exists Q)$   $Q$  output of  $T_1$  and  $Q$  input of  $T_2$
  - Simple, but imprecise
- **Improvement: analyze variables one by one**
  - “ $T_1 \rightarrow_X T_2$ ” iff  $(\exists Q)$  as above and  $Q$  in unit of  $X$
  - Chosen approach



# Global Data-Flow Predicates

*live*( $T_0, X$ ) iff

$(\exists T_0 \rightarrow_X \dots \rightarrow_X T_n)$   
*use\_before\_def*( $T_n, X$ )  
and  
 $(\forall i \in \{1, \dots, n-1\})$   
 $\neg \text{def}(T_i, X)$

Backward fixed point

*available*( $T_n, X$ ) iff

$(\exists T_0 \rightarrow_X \dots \rightarrow_X T_n)$   
*def*( $T_0, X$ )  
and  
 $(\forall i \in \{0, \dots, n-1\})$   
*live*( $T_i, X$ )

Forward fixed point

*reset*( $T, X$ ) iff *available*( $T, X$ ) and  $\neg \text{live}(T, X)$



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# Treatment of Inherited Shared Variables

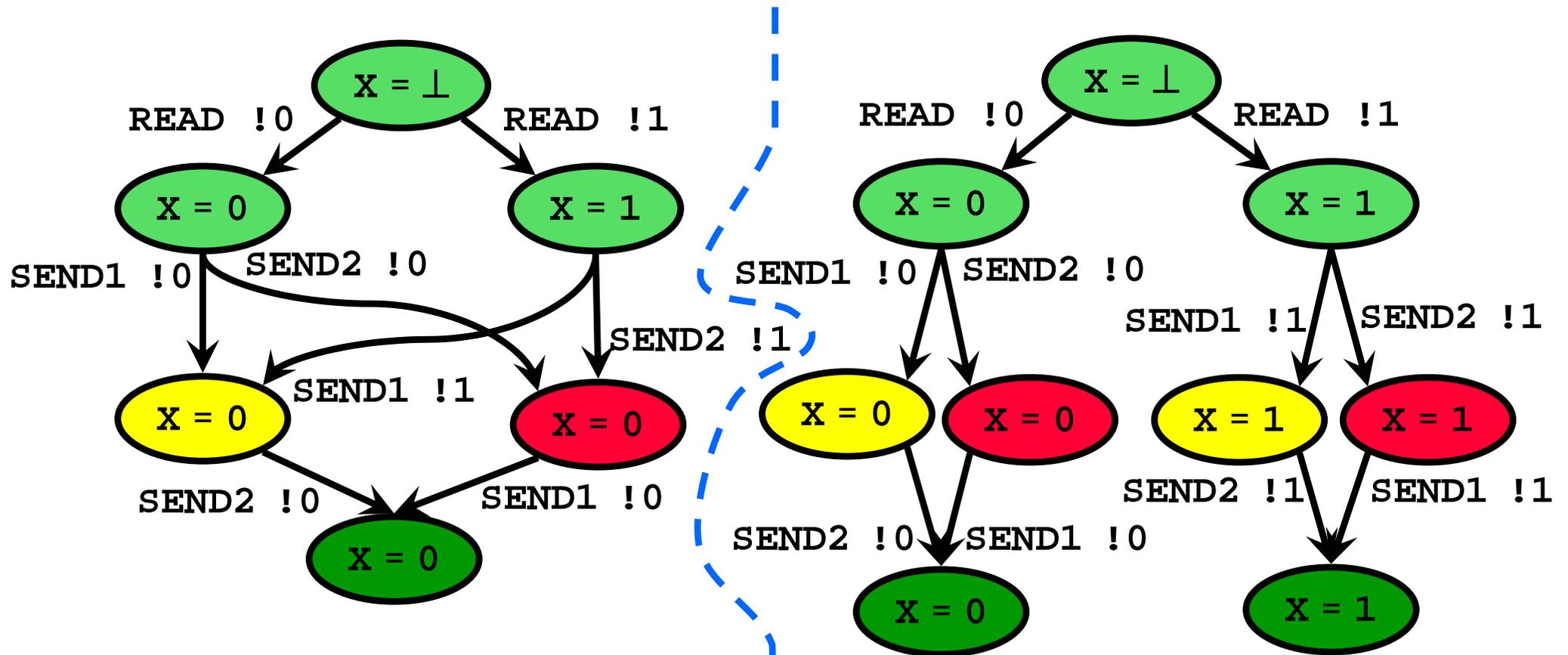
(section 5 of the paper)



# Resetting Shared Variables (1/3)

READ ?X: bit;

(SEND1 !X; stop ||| SEND2 !X; stop)



incorrect graph  
(with resets;  $\perp = 0$ )

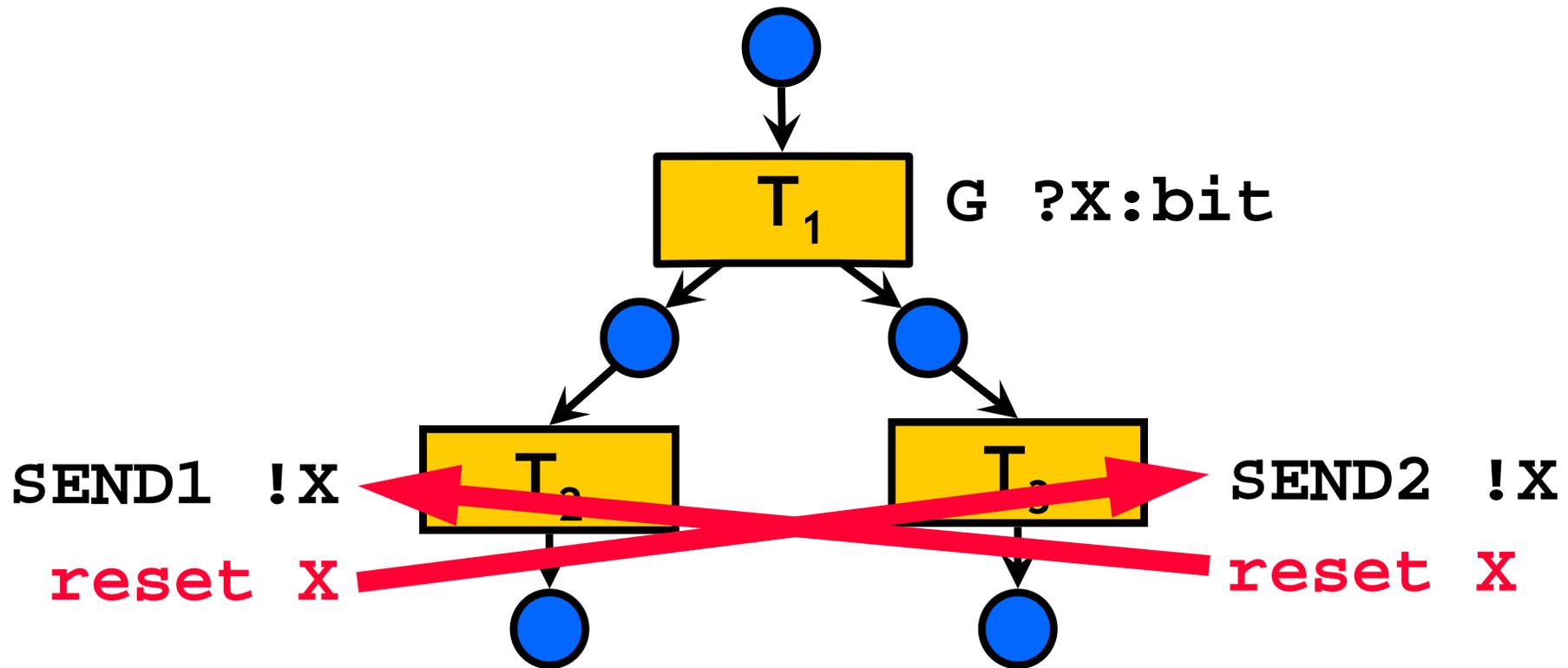
correct graph  
(without resets)



# Resetting Shared Variables (2/3)

READ ?X:bit;

(SEND1 !X; stop ||| SEND2 !X; stop)



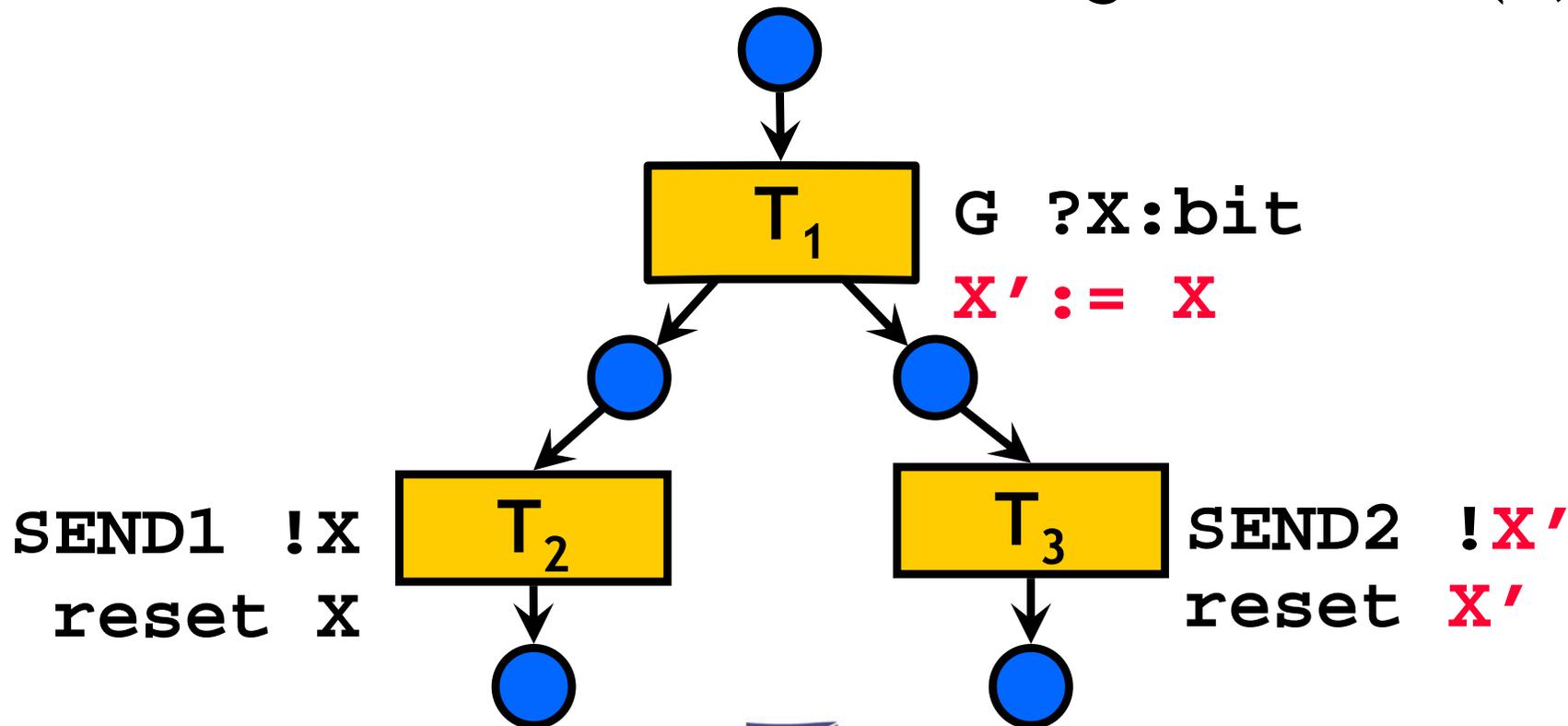
- Without resets, shared variables are read-only
- Inserting resets creates read/write(reset) conflicts



# Resetting Shared Variables (3/3)

## Solution: Duplication of "x" in unit "U"

- Create a new variable  $x'$  attached to U
- Replace  $x$  by  $x'$  in all transitions of U
- Insert " $x' := x$ " in all T entering U s.t.  $Live(T, x)$



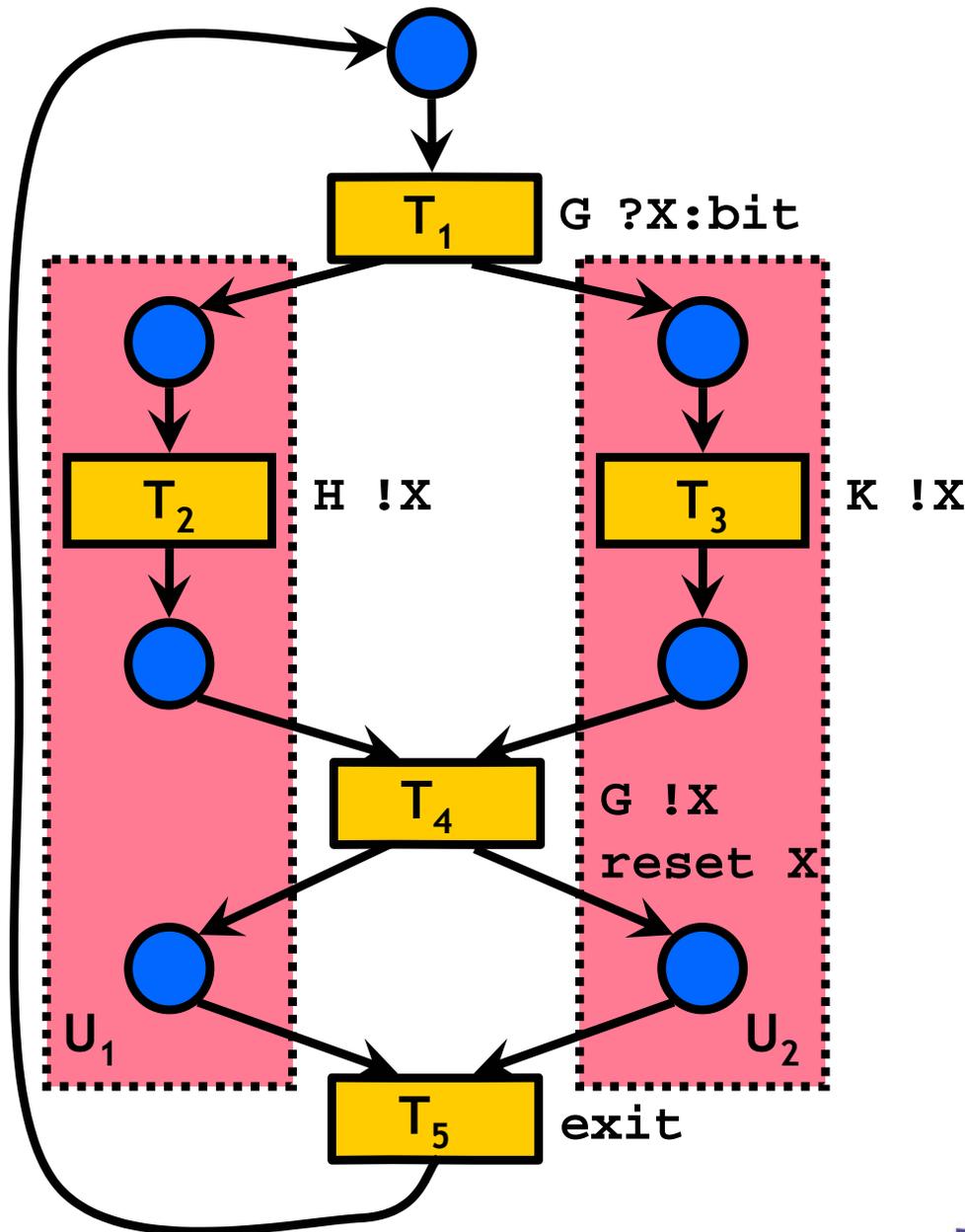
# Which Variables to Duplicate? (1 / 2)

- Variable duplication increases the representation of a state!
- **Goal:** minimal number of duplicated variables
- **Concurrent Units:** “ $U_1 \parallel \parallel U_2$ ”  
 $U_1, U_2$  separate and simultaneously marked
- **Conflict**  $use(T_1, X)$  versus  $reset(T_2, X)$  iff  
 $T_1$  transition of  $U_1$ ,  $T_2$  transition of  $U_2$ , and  $U_1 \parallel \parallel U_2$

**Too rough!**



# Which Variables to Duplicate? (2/2)



- $use(T_2, \mathbf{x})$ ,  $use(T_3, \mathbf{x})$ ,  $use(T_4, \mathbf{x})$
- $reset(T_4, \mathbf{x})$
- $T_2$  transition of  $U_1$
- $T_3$  transition of  $U_2$
- $T_4$  transition of  $U_1$  and  $U_2$
- $U_1 \parallel U_2$

**Conflict  $T_2$  ( $T_3$ ) with  $T_4$ ?**

**NO:**

- $T_4$  synchronizes  $U_1, U_2$
- “ $reset\ \mathbf{x}$ ” in  $T_4$  correct



# Algorithm

compute *concurrent units* and *synchronizing transitions*

$VARs := \{ X_1 \dots X_n \}$

**while**  $VARs$  not empty **do**

choose  $X$  in  $VARs$

**repeat**

compute *local* and *global data-flow*

compute *conflicts*

$U :=$  choose conflicting unit

**if**  $U \neq NULL$  **then**

duplicate  $X$  in  $U$  (yields  $X'$ )

$VARs := VARs \cup \{ X' \}$

**until**  $U = NULL$  (i.e., no more conflicts)

insert “reset  $X$ ” in all  $T$  such that “*reset*( $T, X$ )”



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# Experimental Results

(section 6 of the paper)



# Experimental Results

- Tests: 544 LOTOS value-passing specifications
- State space reduction for 120 examples (22%)
- Average reduction factors  
States: 9 (max 220), Transitions: 12 (max 360)
- 3 examples: generation impossible before reduction factor  $> 10^4$
- Generating all graphs: 4 times faster
- Only 24 examples (4%) requiring duplication
- Increase of state representation outweighed by state space reduction



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# Conclusion

- **Resetting variables in process algebra**
  - Translation to structured Petri nets with variables
  - Local and global data-flow analysis
- **Rich model: Hierarchy of nested processes**
- **Tests on 544 examples: reductions up to  $10^4$**

## Open issues

- **Unrestricted creation/destruction of processes**
- **Handling of shared read-write variables**

