Distributed LTL Model-Checking

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Enumerative LTL Model-Checking

Automata Approach – Basic Principle

- The LTL model-checking problem "$A \models \varphi$?" is reduced to

  is the language recognized by $A \times B_{\neg\varphi}$ empty ?

- BA C can be represented as a graph G_C
- $L(C)$ is non-empty iff G_C has a reachable accepting cycle

Graph problem:

Given: Digraph with a source vertex and subset of vertices marked as accepting.

Question: Does there exist a cycle which contains at least one accepting vertex and is reachable from the source ?

In positive case generate generate the cycle and a path to it from source.

Distributed LTL Model-Checking

Platform

- Network of workstations (NOWs).
- No shared memory (combined memory).
- Communication by message passing.

Graph distribution

- Graph given implicitly by $(F_{init}, F_{successor})$
- Distributed data – **partition function** assigns vertices to workstations

Graph problem: Detection of a reachable accepting cycle in a distributed graph.

Distributed Algorithms

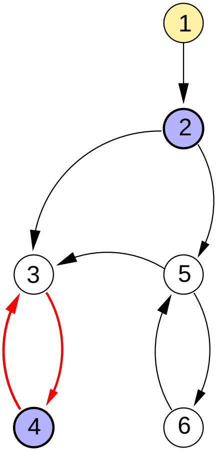
- new algorithms needed
  - sequential solution: postorder – difficult to parallelize (PTIME)
  - parallel solution: reachability – efficient parallelization (NC)
- **travel & propagate (repeated reachability)**

Four groups – Six algorithms

- BFS instead of DFS
  [Maximal Predecessors, Back-Level Edges]
- SCC-based approaches
  [Elimination of SCCs – forth and back]
- Reduction to another problem
  [Negative Cycles]
- Additional data
  [Dependency Structure]

Maximal Accepting Predecessors

[Brim, Černá, Moravec, Šimša – FMCAD 2004, PDMC 2005]



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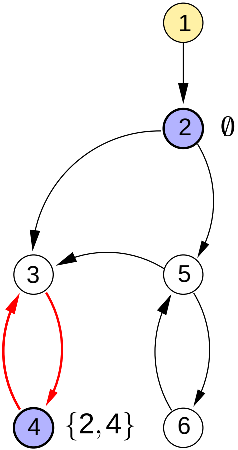

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Algorithm

forall $s \in A$ **do**
  $Acc(s)$ = set of accepting
    predecessors of s **od**
forall $s \in A$ **do**
  if $s \in Acc(s)$ **then return** CYCLE
od
return NO CYCLE

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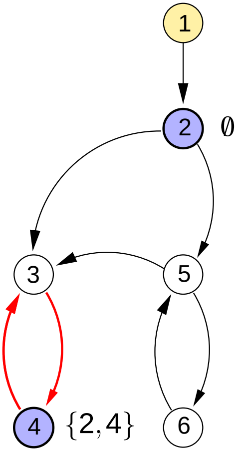

Idea

Each accepting vertex on an accepting cycle is its own predecessor.

- Storing all predecessors is expensive.
- Order accepting vertices and store **the maximal one** only.

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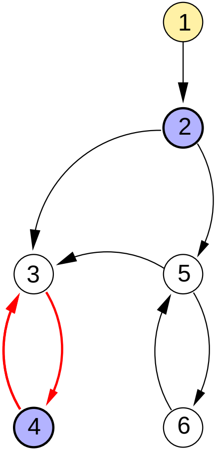

Improved idea

If an accepting vertex is the maximal accepting predecessor of itself, then it belongs to an accepting cycle.

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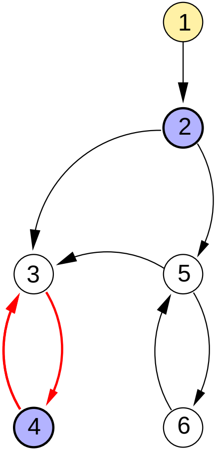

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Algorithm

while $A \neq \emptyset$ **do**
compute *map*; { max. accepting predecessors }
if $(\exists u \in A : map(u) = u)$
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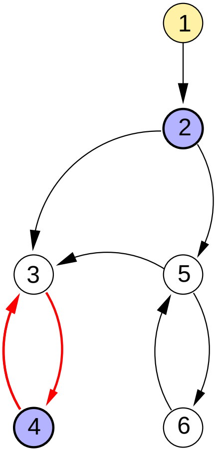

Ordering

$4 > 2 > \bot$

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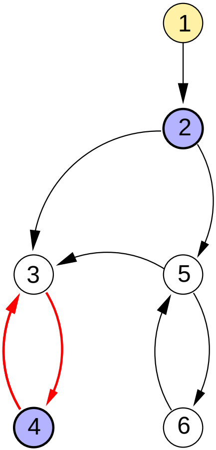

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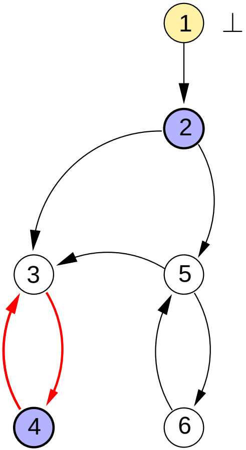

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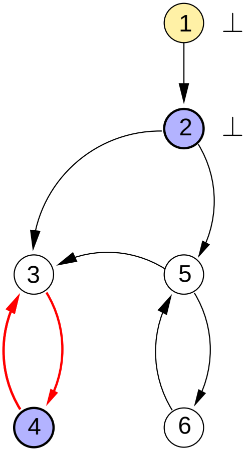

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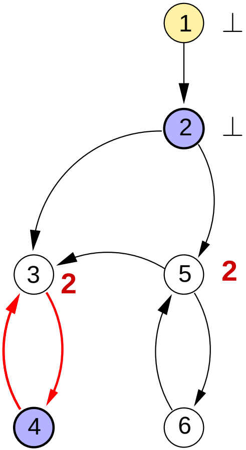

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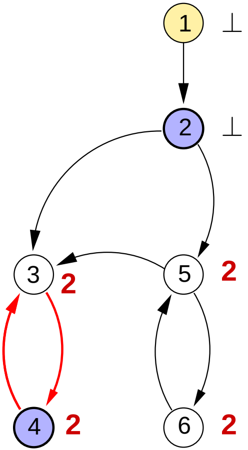

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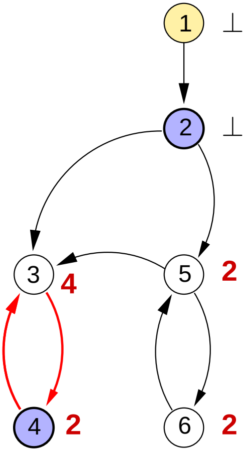

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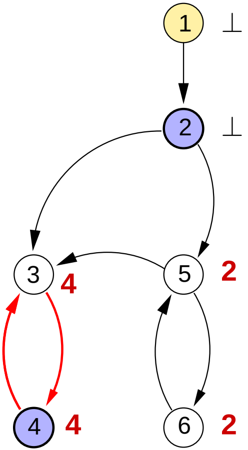

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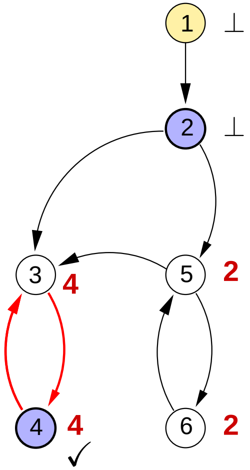

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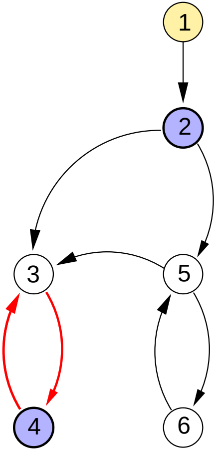

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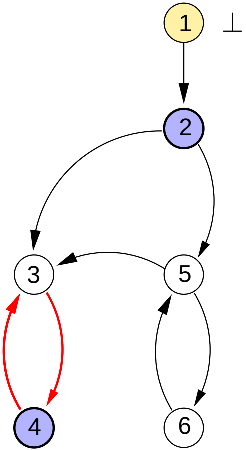

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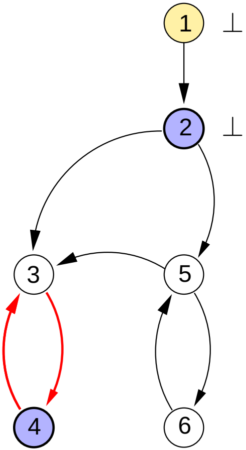

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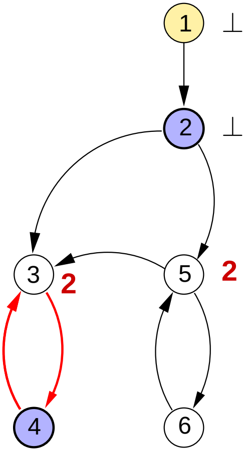

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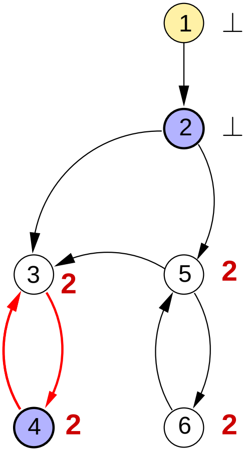

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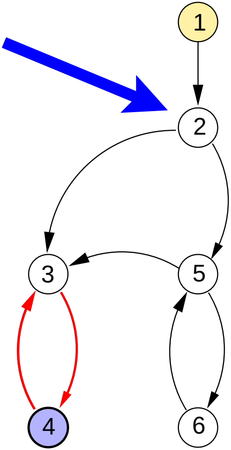

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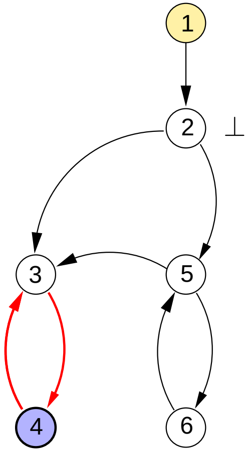

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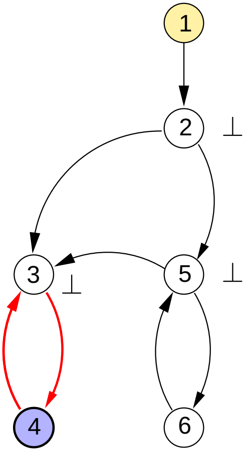

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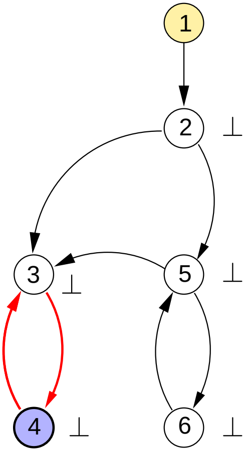

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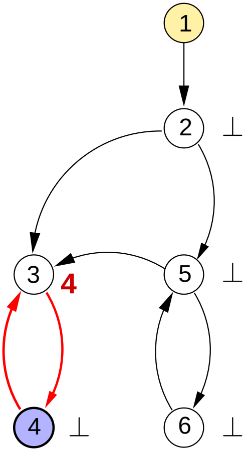

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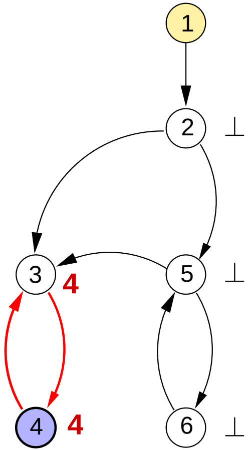

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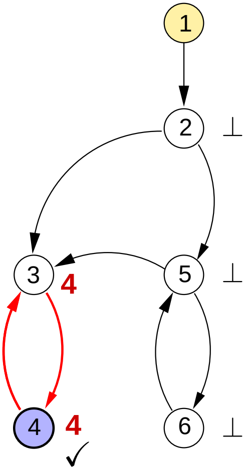

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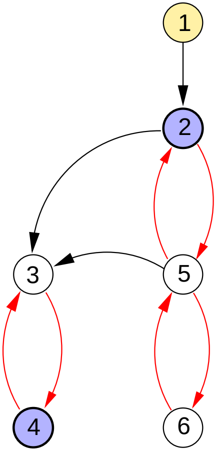
Maximal Accepting Predecessors

Comments

- An accepting cycle in G can be formed from vertices with the same maximal accepting predecessor only.
- A graph induced by the set of vertices having the same maximal accepting predecessor is called predecessor subgraph.
- Every cycle in the graph is completely included in one of the predecessor subgraphs.
- Re-computing the MAP function can be done in parallel for every predecessor subgraph.
- DFS gives optimal ordering – heuristics for “good” ordering.

Back-Level Edges Algorithm

[Barnat, Brim, Chaloupka – ASE 2003]



Back-Level Edge

Destination state has no greater distance from source vertex than its source state.

Back-Level Edges Algorithm

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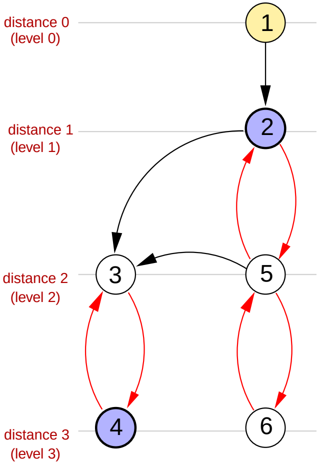

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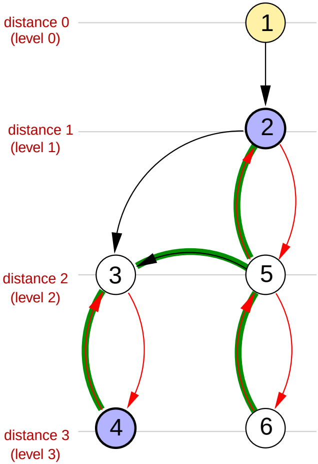

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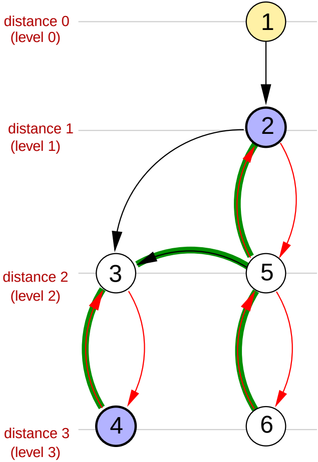

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Each cycle must contain a back-level edge.

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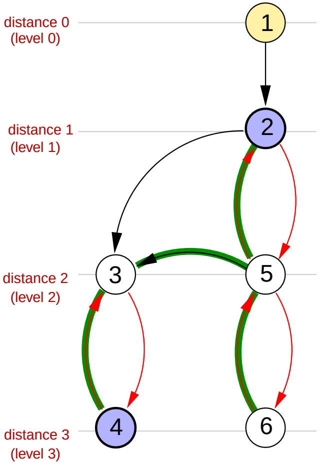

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Algorithm

- Discover all back-level edges – level synchronized BFS.
- Check if there is an edge that is part of a cycle (nested procedure).

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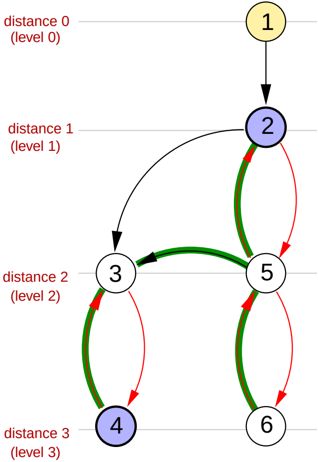

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Algorithm

for level = 0 **to** . . . **do**
  L = all current BL edges
  forall $(s,t) \in L$ **do in parallel**
    test_cycle(s,t,$| L |$)
  od
od

proc test_cycle(s,t,$| L |$)
  propagate *s*
  if *s* propagated to itself **then** ✓
  else if current BL passed $> | L |$ **then** ✓

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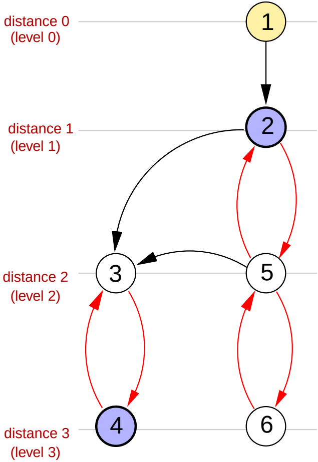

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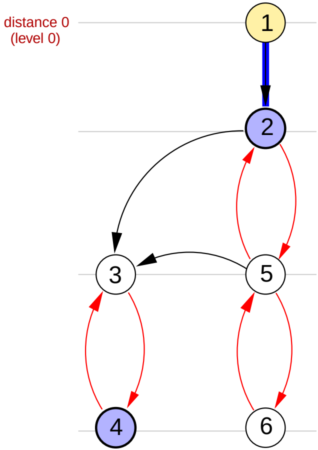

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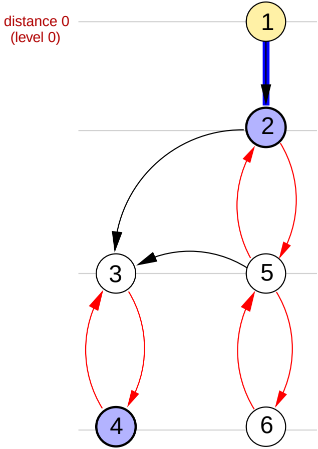
Back-Level Edges Algorithm

[Barnat, Brim, Chaloupka – ASE 2003]



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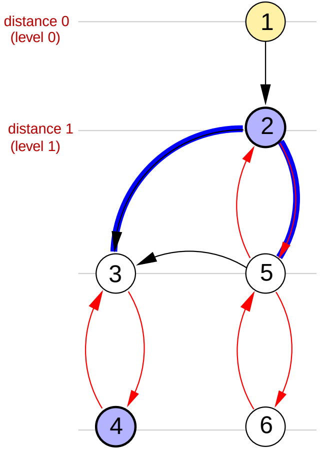

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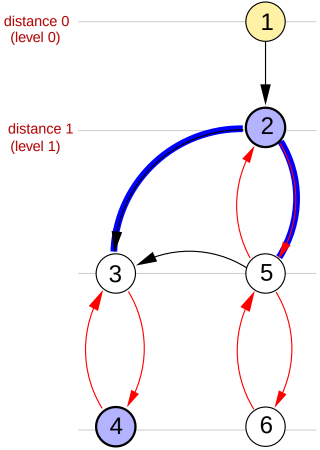

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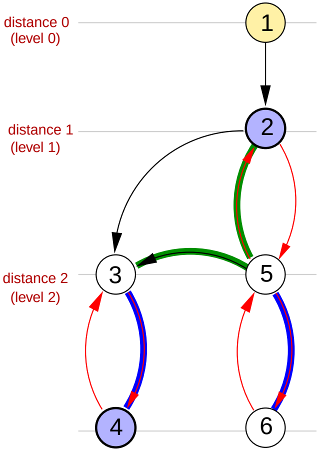

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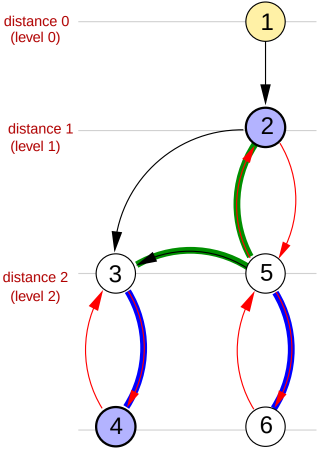

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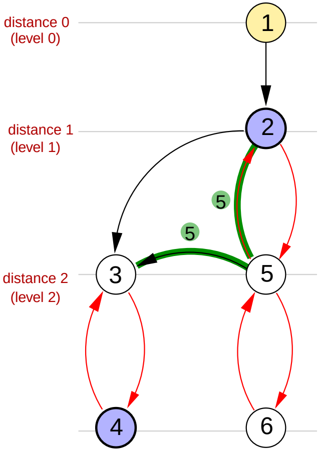

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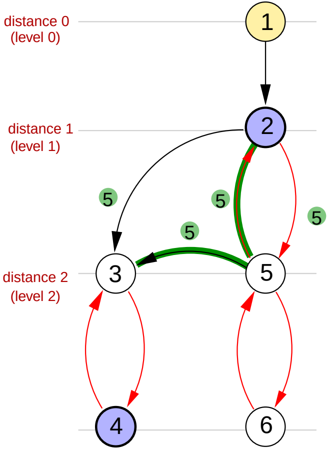

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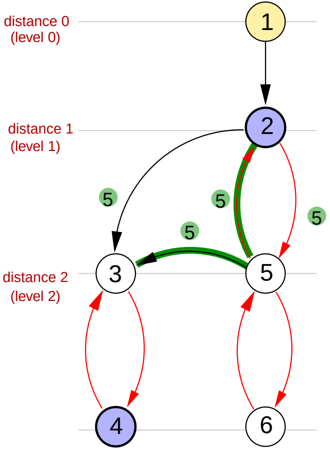

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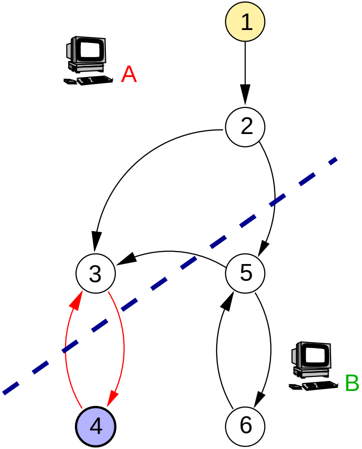

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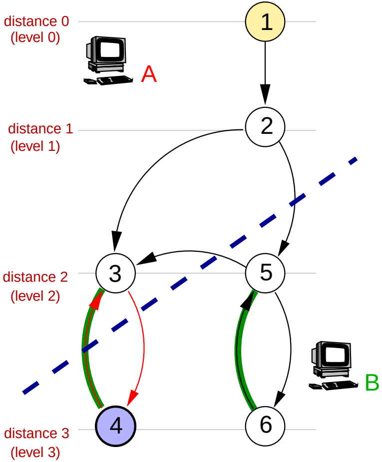

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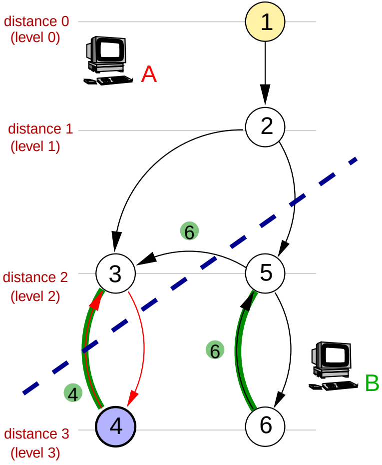

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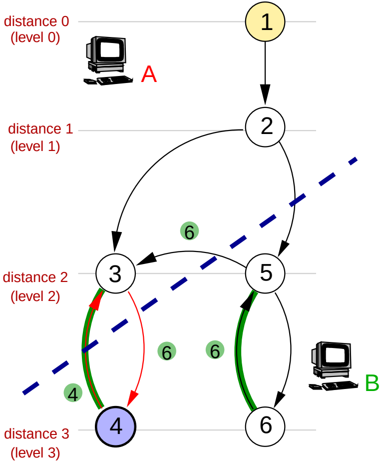

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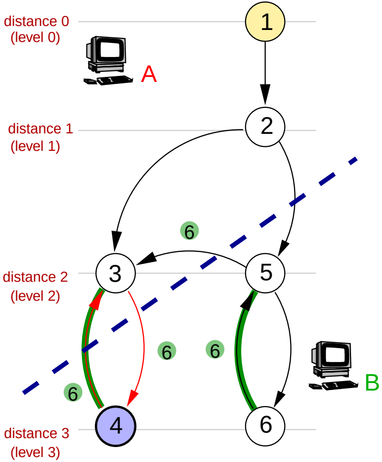

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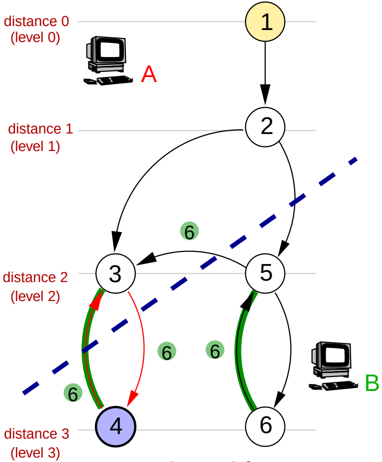

stops after #bl edges

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Back-Level Edges Algorithm

Comments

- Accepting cycle detection (additional bit required)
- Partial Order Reduction

Partial Order Reduction

- Exploring subsets of successors of states (ample sets)
- Conditions ensuring correctness: C0 – C3
- C3-DFS: at least one fully explored state on each cycle
- C3-BFS: **Full expansion of source states of back-level edges**

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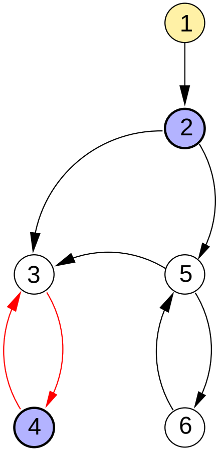
SCC-Based Algorithm

[Černá, Pelánek – SPIN 2003]



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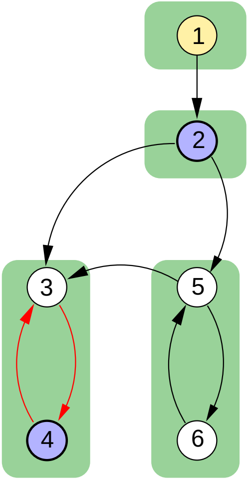

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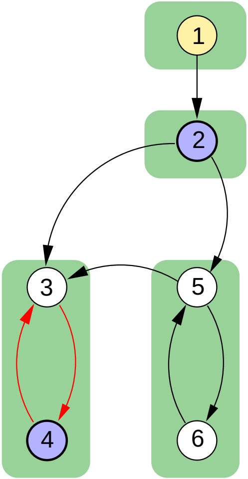

Idea

Each reachable accepting cycle is contained in a nontrivial strongly connected component which is reachable from the source vertex and contains an accepting vertex.

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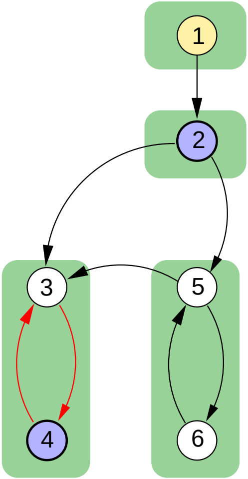

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Remove all SCCs **without** required properties.

- remove trivial SCCs
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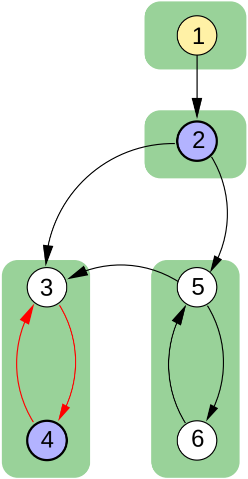

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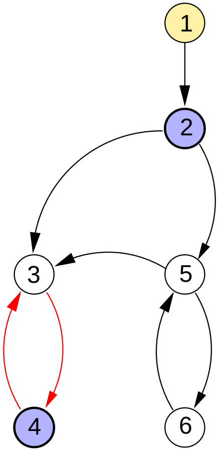

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Algorithm on vertices

while not finished **do**

- remove vertices which are not reachable from accepting vertices
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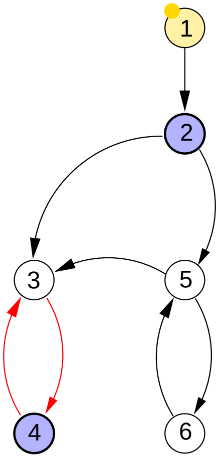

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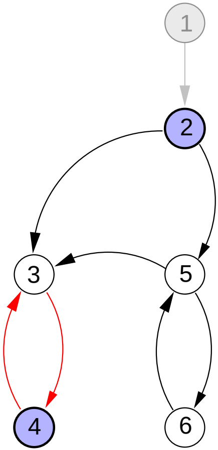

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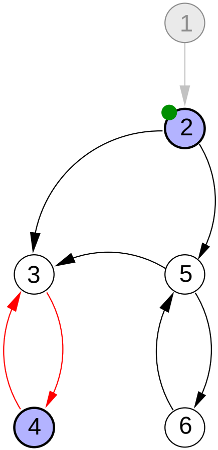

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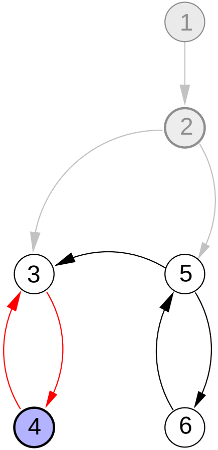

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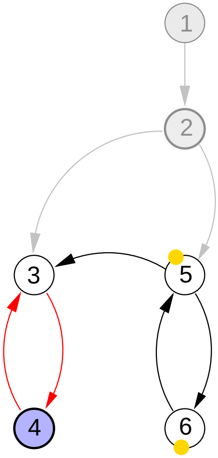

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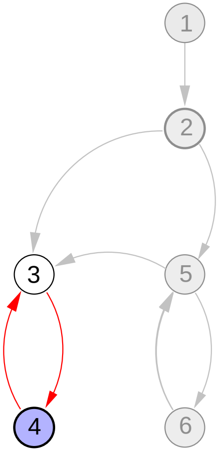

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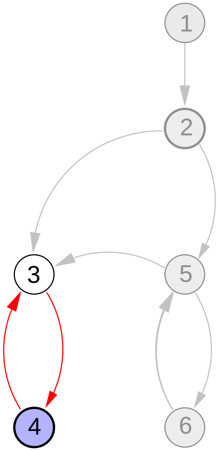

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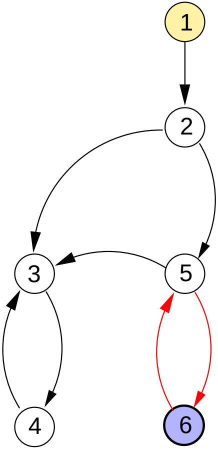

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Main idea is the same: Each accepting cycle is contained in a nontrivial strongly connected component which is reachable from the source vertex and contains an accepting vertex.

Computing successors may be expensive.

Store edges and check symmetric conditions using predecessors.

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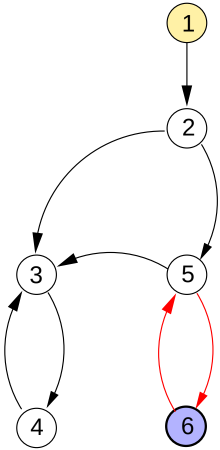

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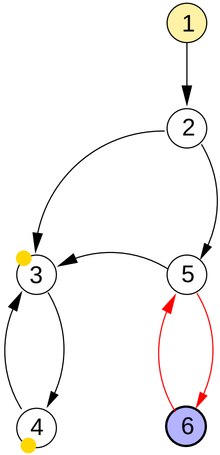

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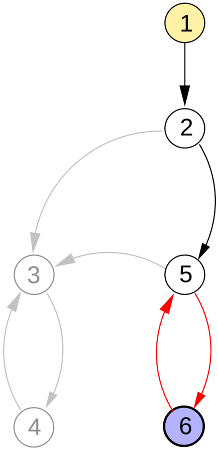

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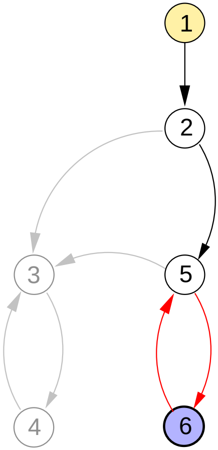

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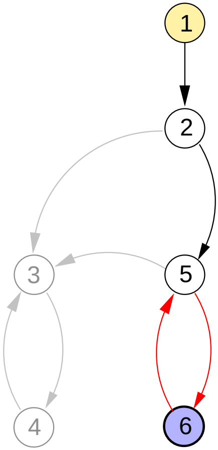

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SCC-Based Algorithms

Comments

- Time complexity is $O(h.(n+m))$
  - n - number of vertices
  - m - number of edges
  - h - height of SCC quotient graph
- Almost linear complexity
- Only one external iteration for weak BA graphs
- Algorithm does not work on-the-fly

Negative Cycles Algorithm

[Brim, Černá, Krčál, Pelánek – FSTTCS 2001]

Idea

- Reduce BA emptiness problem to another one which can be distributed more easily.
- **Detecting negative cycles in the SSSP problem.**

Negative cycles coincide with accepting cycles.

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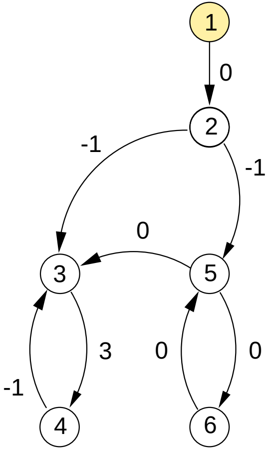

SSSP

For each vertex compute the smallest distance from source a build the parent graph (tree)

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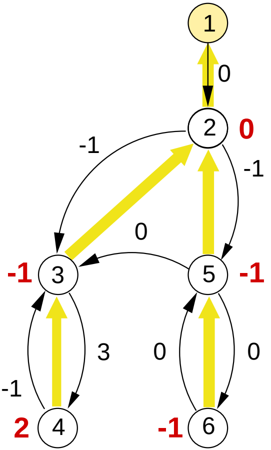

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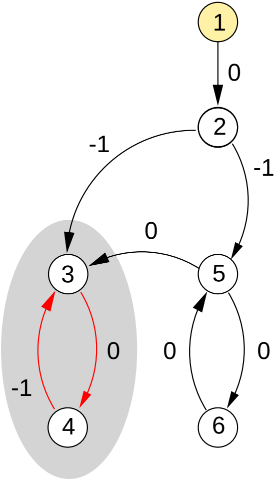

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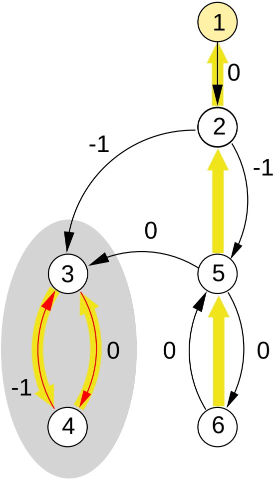

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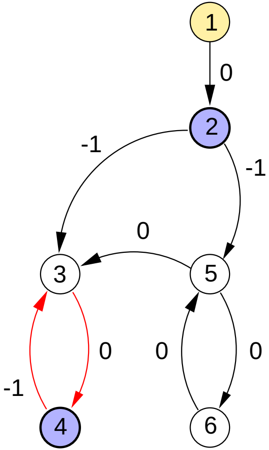
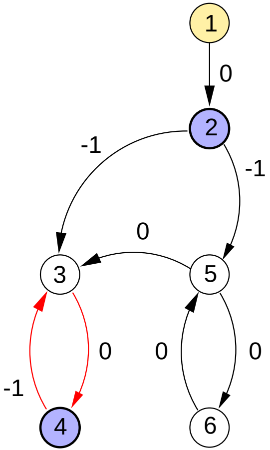


Idea

Reduction: Assign **-1** to out-going edges of accepting vertices, otherwise assign **0**.

Negative Cycles Algorithm

[Brim, Černá, Krčál, Pelánek – FSTTCS 2001]



Idea

Reduction: Assign **-1** to out-going edges of accepting vertices, otherwise assign **0**.

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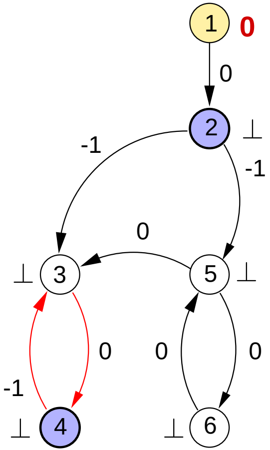

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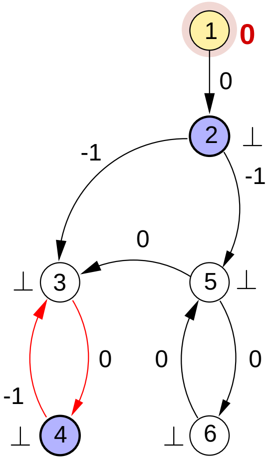

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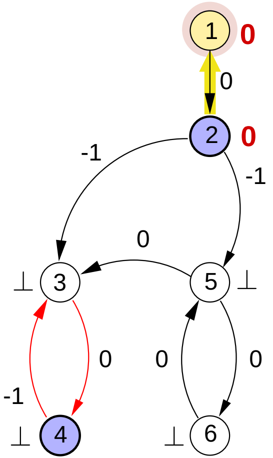

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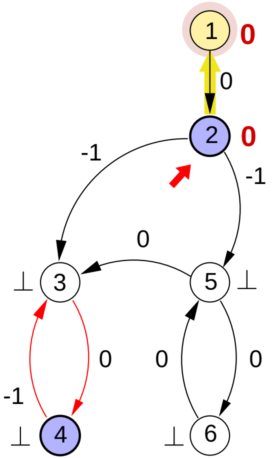

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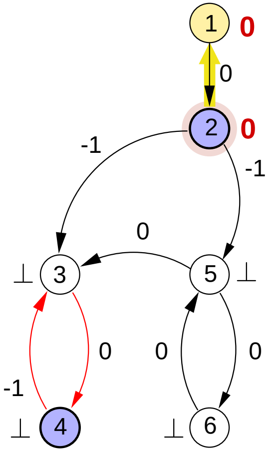

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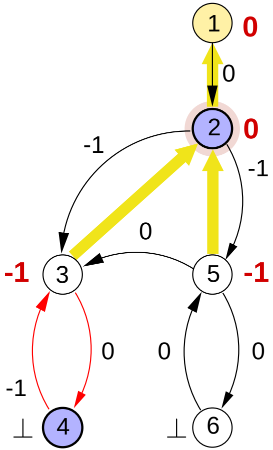

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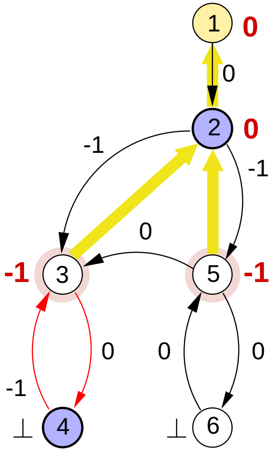

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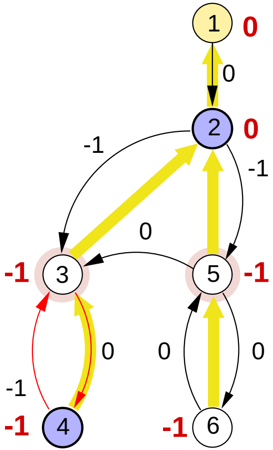

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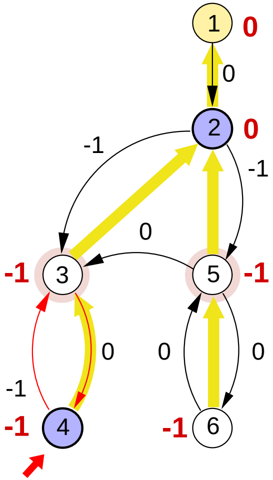

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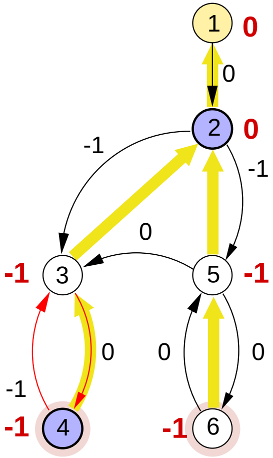

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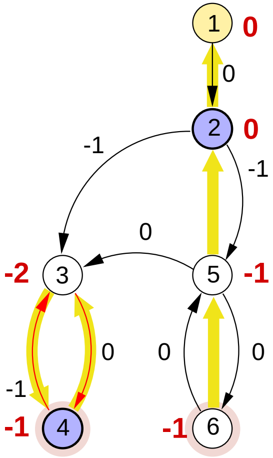

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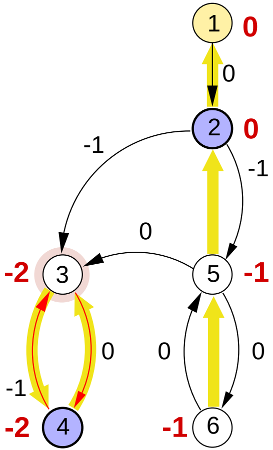

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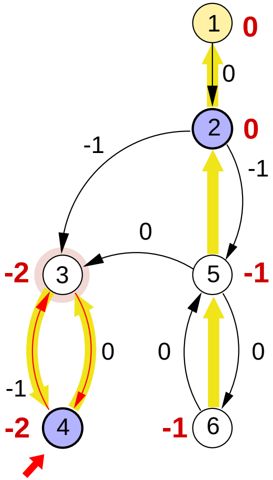

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Comments

- Strategies to detect presence of a negative cycle
  - time out
  - **walk to root (WTR)**
  - subtree traversal
  - **amortized search**

– time complexity is $\mathcal{O}(n^3/P)$
P - number of processors, n number of vertices

\+ algorithm is comparable with nested-DFS algorithm on all graphs

\+ algorithm is significantly better on graphs **without accepting cycles**

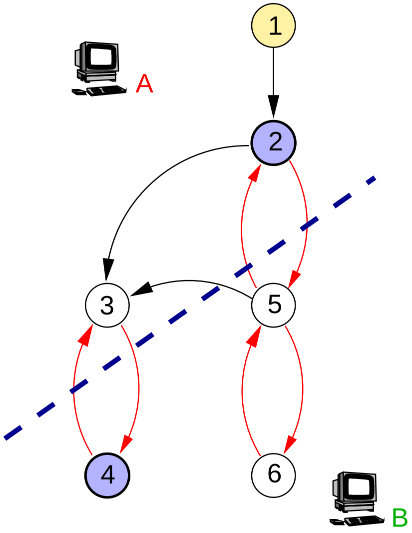
Dependency Structure Algorithm

[Barnat, Brim, Stříbrná - SPIN 2001]

Idea

- Replace the graph by another smaller one:
  - Border vertices and accepting accepting only
  - Edges represent reachability (dependency) among these vertices.
- There is an accepting cycle in dependency graph iff there is a splitted accepting cycle in the original graph.
- Dependency graph is on-the-fly and is distributed as well.

Dependency Structure Algorithm – Example



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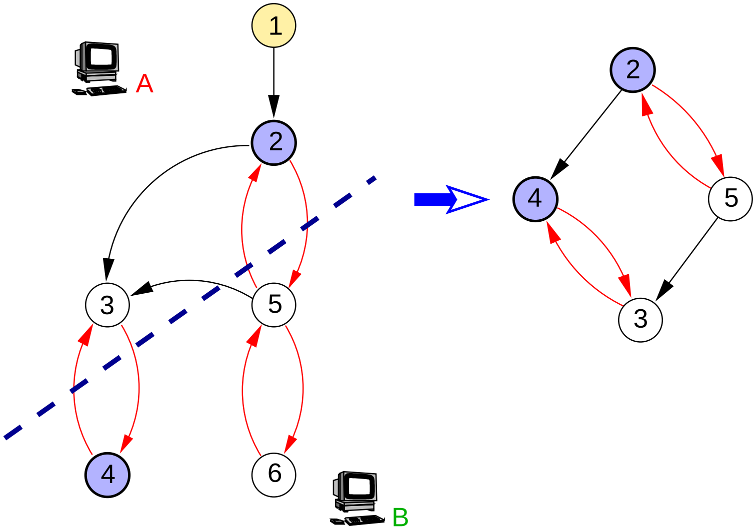

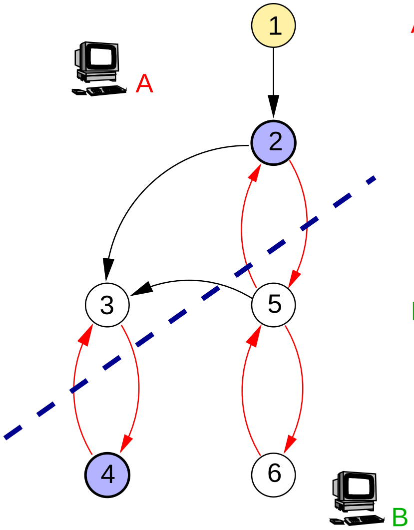

A

2 → 4

2 → 5

3 → 4

B

4 → 3

5 → 3

5 → 2

Dependency Structure Algorithm

Comments

- Dependency structure:
  - Each workstation maintains its own local dependency structure.
  - Dynamic – vertices are added and removed.
- Additional memory required:
  ($O(n.r)$ on average, where r is the maximal out-degree and n is the number of states)
- Any distributed cycle detection algorithm can be used.

Conclusion

- Core problem of automata based LTL model-checking is the detection of reachable accepting cycles in the state space.
- Alternative approaches to distributing LTL Model-Checking presented.
- All algortihms implemented in DiVinE.
- Parallelization is used because parallel systems are complex and their development is difficult – development of parallel algorithms for their analyzis is mentally and technically challenging as well.
- Work in progress:
    - Extension to GBA, RA, SA.
    - LTL MC of probabilistic and real-time systems.
    - Cost analysis.