Experiences with Cluster-Based Model Checking of Stochastic Systems

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Overview

- Background
- Cases and machinery
- Generation
- Solution
- Concluding remarks

Background: High-level specifications

- man-made discrete systems can be described well using high-level specification languages:
  – communication protocols, distributed systems (process algebras)
  – computer systems, manufacturing systems (Petri nets)
- not all system properties can be verified directly from the high-level specification
- many properties can only be verified using the large underlying STS
- derivation of STS is a time-consuming task
- important aspect of "model checking"

Background: Adding (stochastic) time

- model specification is enhanced with stochastic timing information
- underlying STS can be interpreted as a *continuous-time Markov chain* $\cdots$ which can be solved numerically
- various model specification techniques can be imagined:
  e.g., stochastic process algebras, or *stochastic Petri nets*
- variety of logic-based property specifications possible

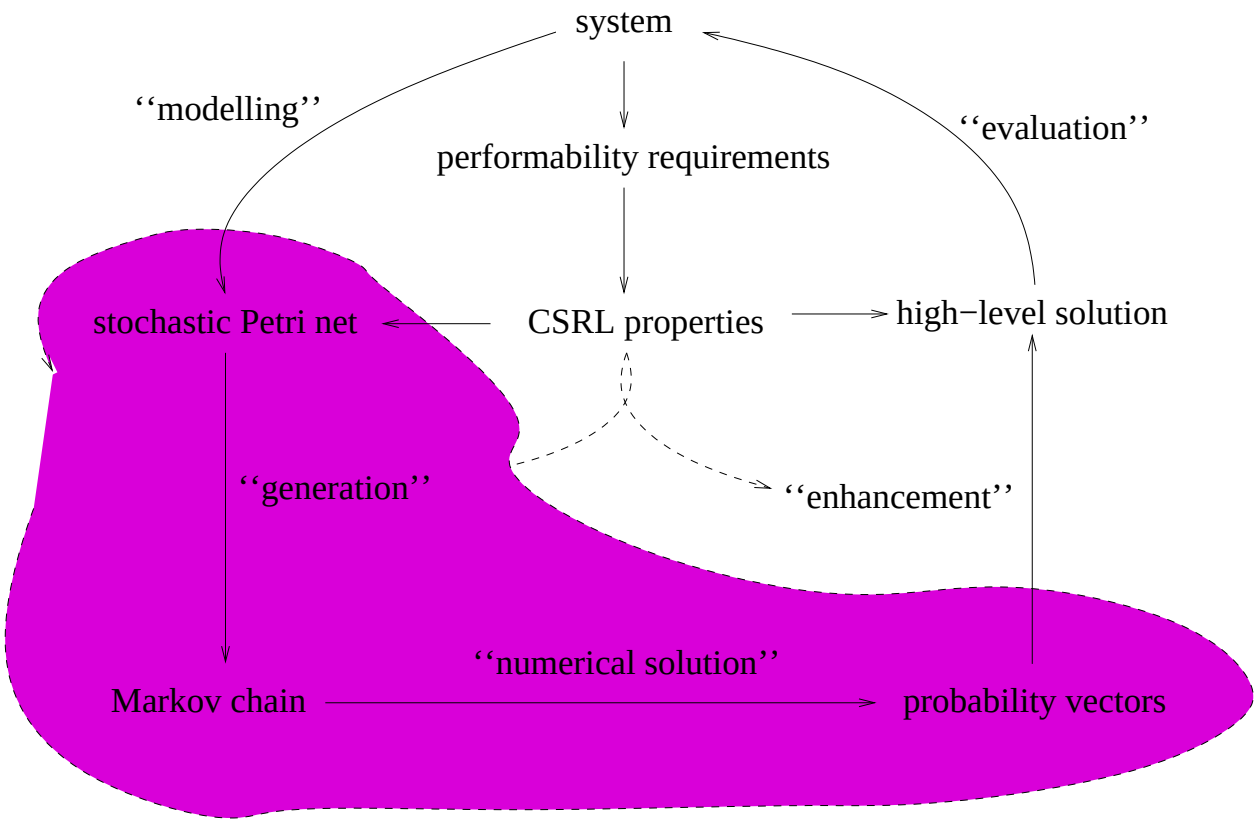
Hence, we combine...

- aspects of CTL-type model checking
- with aspects of numerical analysis of Markov chains

$\Rightarrow$ **CSL and CSRL model checking of Markovian models**

- try to combine best techniques of both worlds!
- in any case: computational and storage requirements are enormous!

Modelling & evaluation cycle



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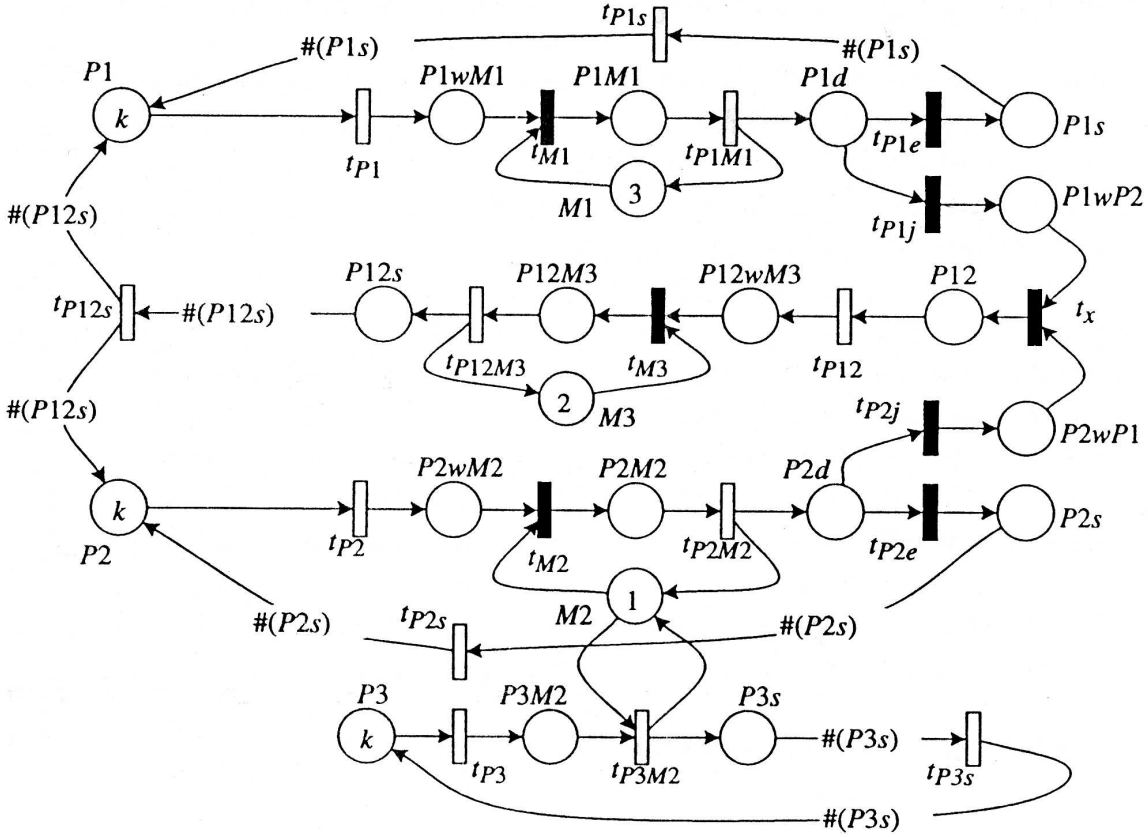
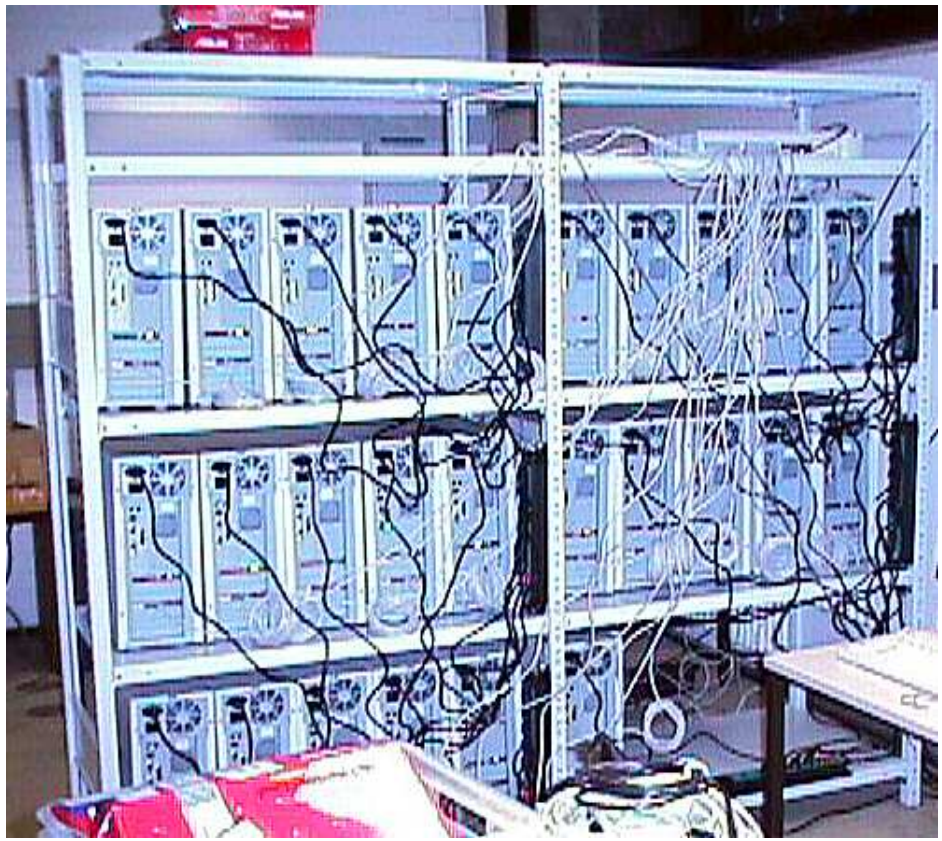
Cases and Machinery: the FMS case (I)



Cases and Machinery: the FMS case (II)

pallets k	states n	arcs$a$	a/n	a/n^2
1	54	155	2.87	5.31e-02
3	6 520	37 394	5.73	8.80e-04
5	152 712	1 111 482	7.28	4.77e-05
7	1 639 440	13 552 968	8.27	5.04e-06
9	11 058 190	99 075 405	8.96	8.10e-07
11	54 682 992	518 030 370	9.47	1.73e-07
13	216 427 680	2 611 411 257	12.07	5.57e-08
15	724 284 864	9 134 355 680	12.61	1.74e-08

Cases and Machinery: Cluster at RWTH



Cases and Machinery: From DAS-2 to DAS-3

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STS generation: Three possibilities

- explicitly, serially
- explicitly, distributed
- implicitly

Serial, explicit STS generation

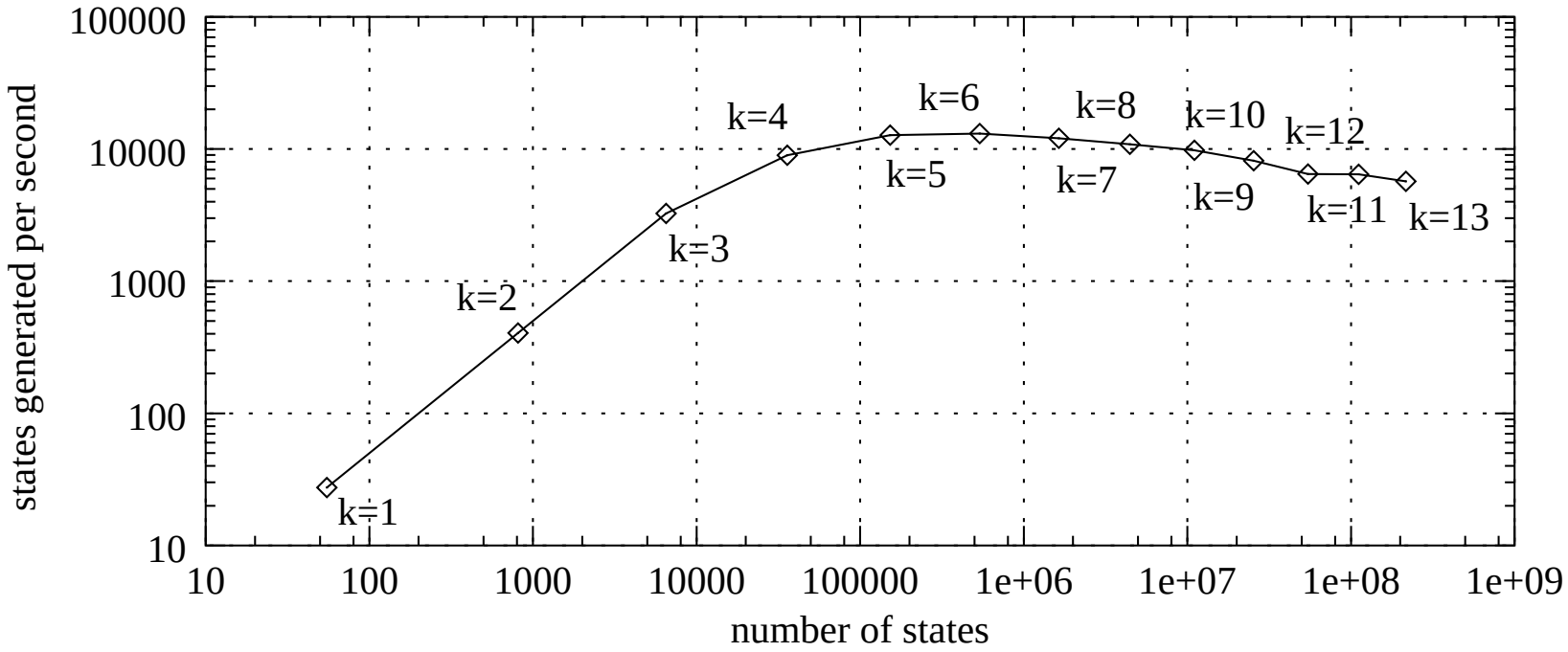
- standard tree-based search algorithm

- key issues:
  - 1: data structure for state space
  - 2: data structure for intermediate states
  - 3: next state computation (enabled(s)?)
  - 4: does state s exist?

- choices:
  - 1: hash table: open addressing with double hashing
  - 2: stack with top in main memory, all of body on disk
  - 3, 4: depends strongly on high-level model (SPNs: good!)

- transition relation directly stored on disk

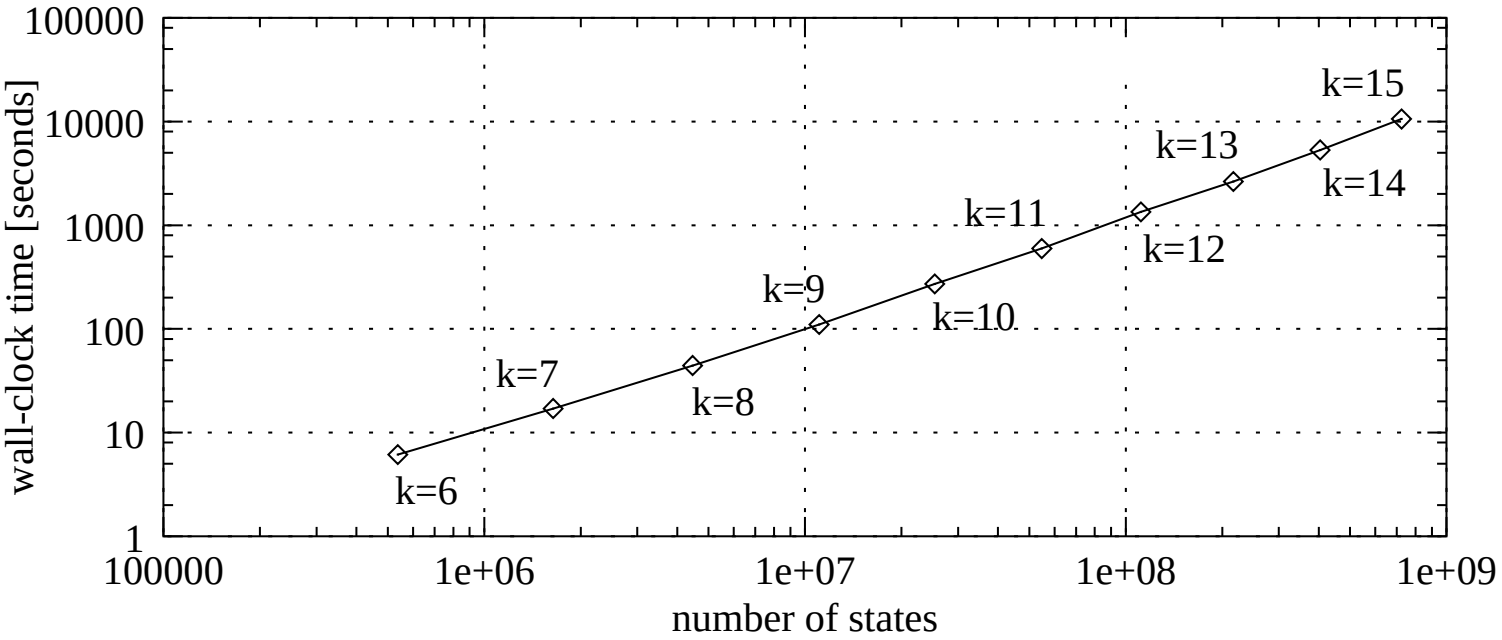
Serial STS generation speed for FMS(k)



Distributed, explicit STS generation

- use an extra hash function to map states onto processors (nodes)
- find good:
  – state balance: equal number of states per node
  – while avoiding cross-arcs = minimizing communication
- good, balanced solutions do exist
- divides state space and transition relation over all available nodes
- shows very good speed-ups

26-node distributed explicit STS generation for FMS(k)



Implicit generation

- use of BDDs, MTBDDs or MxDs (matrix diagrams)
- extremely fast even on single nodes, for many millions of states
- the next step, i.e., model checking or Markov chain solution, is much more computationally intensive

$\Rightarrow$ no work on parallellisation required here, unless state space reduction is performed on-the-fly

- better concentrate on the true memory, communication and computation bottlenecks!

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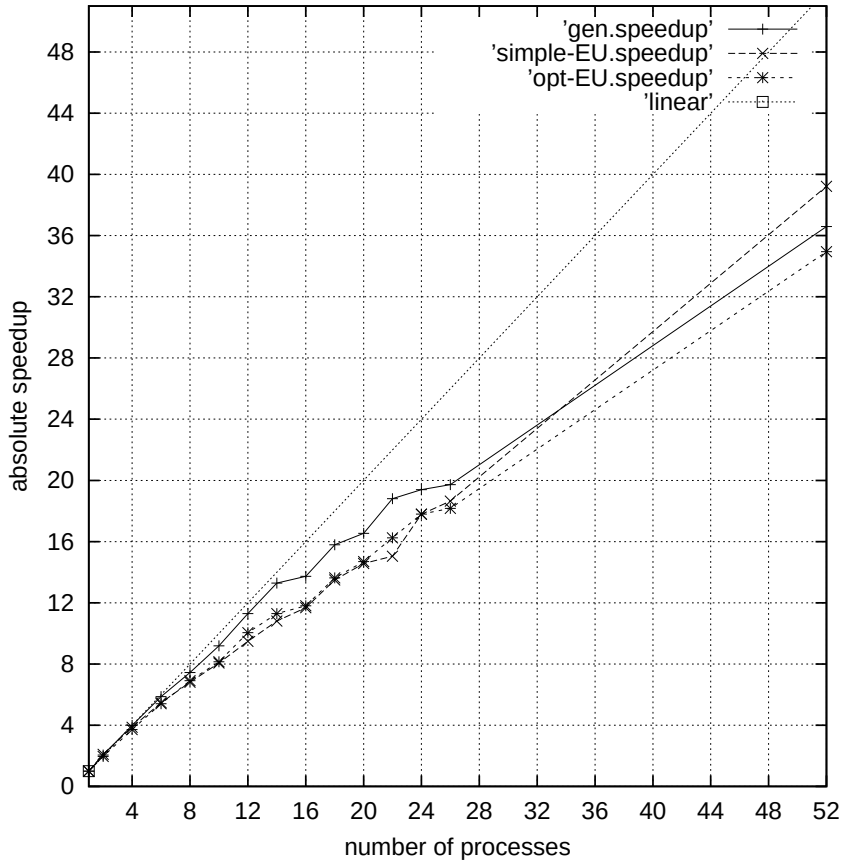
What does "solution" mean?

- CTL model checking of Markov chain interpreted as STS
- CSL and CSRL model checking, meaning, the solution of large equation systems

Distributed explicit CTL model checking of Petri nets

- generalised the algorithms for STS generation
- for all the operators of CTL (E[X.], E[.U.], A[.U.])
- do need the state space in some form readily available
- but recompute the transition relation on the fly
  (both backward and forward, depending on the CTL query)
- found very good speed-ups and efficiencies

Kanban model with $11\,261\,376$ states; S-EU and O-EU



The other type of STS "solution"

- the underlying STS is a Markov chain now, from which we want to compute probabilistic information
- the CSL semantics contains two important operators:
  – steady-state operator: requires steady-state probabilities
  – probability operator: requires transient-state probabilities
- both operators require series of matrix-vector multiplications to be performed
- hence, doing MVMs quickly, serially or distributed, is the key issue!

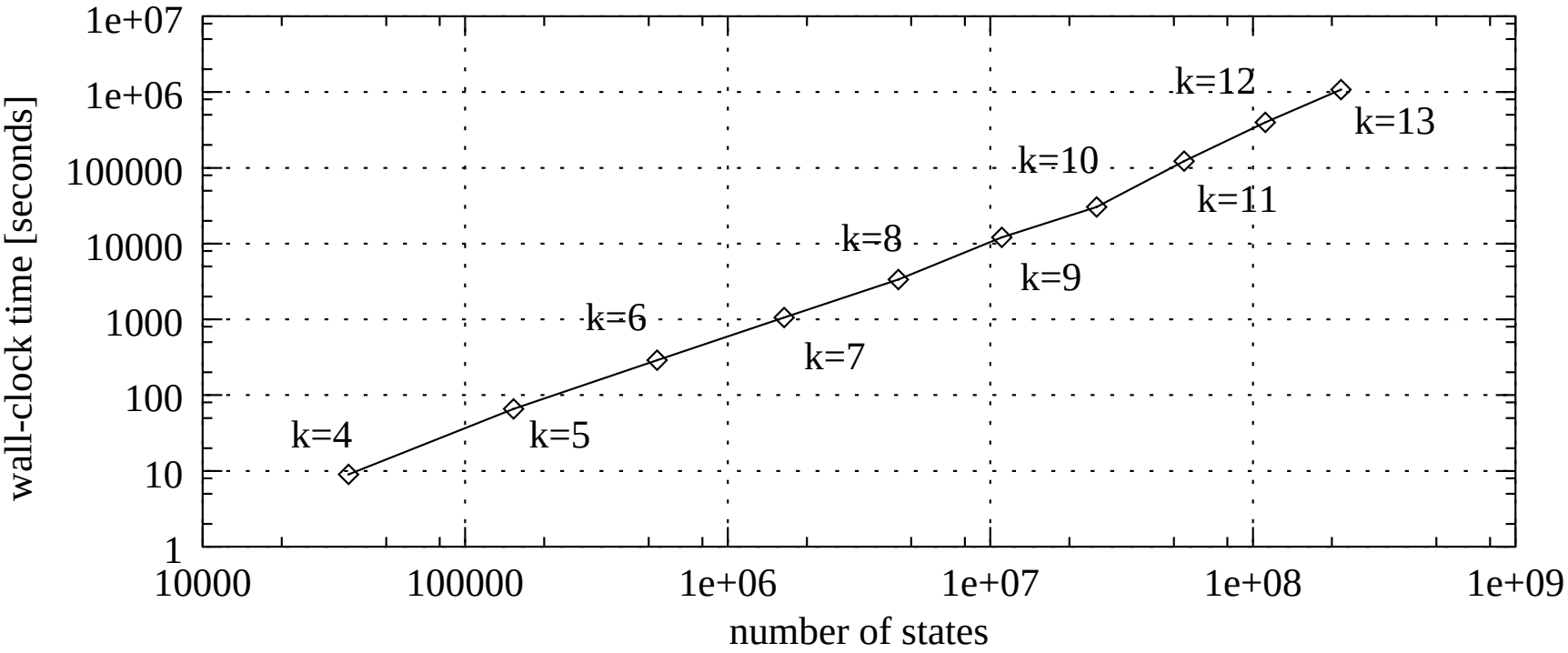
Steady-state solution (I)

- solution of $\underline{\pi} \cdot \mathbf{Q} = \underline{1}$, with $\sum_i \pi_i = 1$
- only iterative methods can be used: $\underline{x}^{(i+1)} := \underline{x}^{(i)} \cdot \mathbf{Q}^*$
  - Jacobi: simple, slow method, 2 iteration vectors
  - Gauss-Seidel: faster, 1 iteration vector
  - Conjugate Gradient Squared; requires more iteration vectors
- vector $\underline{\pi}$ is **very big** but must be **in-core**, otherwise too slow
- storage of $\underline{\pi}$ is a key problem
- **out-of-core** or implicit matrix $\mathbf{Q}^*$; it does not fit in main memory!

Serial steady-state solution FMS(k)

k	Jacobi				CGS			
	$\epsilon = 10^{-6}$		$\epsilon = 10^{-9}$		$\epsilon = 10^{-6}$		$\epsilon = 10^{-15}$	
	steps	time	steps	time	steps	time	steps	time
1	260	0:00:00	536	0:00:00	32	0:00:00	230	0:00:00
2	212	0:00:00	473	0:00:00	85	0:00:00	517	0:00:00
3	312	0:00:00	682	0:00:01	137	0:00:01	849	0:00:05
4	402	0:00:09	889	0:00:20	181	0:00:09	1465	0:01:13
5	491	0:01:02	1105	0:02:19	236	0:01:06	2424	0:11:40
6	584	0:05:51	1344	0:13:27	255	0:04:50	2202	0:52:12
7	686	0:18:33	1589	0:43:00	309	0:17:39	3981	3:43:53
8	784	1:03:24	1829	2:27:58	338	0:56:00	2905	8:03:47
9	881	3:15:45	2073	7:40:50	392	3:08:03	—	—
10	980	10:30:59	2366	25:23:24	373	8:27:45	—	—
11	1080	48:29:32	2734	122:45:26	363	34:10:40	—	—

Serial steady-state solution FMS; CGS(4–11), J(12–13); $\epsilon = 10^{-6}$



Distributed steady-state solution: basics

- solution of $\underline{\pi} \cdot \mathbf{Q} = \underline{1}$, with $\sum_i \pi_i = 1$
- only iterative methods can be used: $\underline{x}^{(i+1)} := \underline{x}^{(i)} \cdot \mathbf{Q}^*$
  - Jacobi: simple, slow method, 2 iteration vectors, easy to parallellise
  - Gauss-Seidel: faster, 1 iteration vector, very difficult to parallellise
  - CGS: requires more iteration vectors, but good to parallellise
- **out-of-core**: the matrix $\mathbf{Q}^*$ does not fit in main memory, but can be partitioned over many nodes
- solution vector $\underline{\pi}$ can be distributed over many nodes

Distributed steady-state solution: splitting the work

- each processor stores part of $\mathbf{Q}^*$ (disk) and computes part of $\underline{\pi}$ (memory)
- non-local probabilities are explicitly requested (non-blocking send and receive) from other nodes (for 2 nodes):

$$(\underline{\pi}_1, \underline{\pi}_2)^{\text{new}} := (\underline{\pi}_1, \underline{\pi}_2)^{\text{old}} \cdot \begin{pmatrix} \mathbf{Q}^*_{11} & \mathbf{Q}^*_{12} \\ \mathbf{Q}^*_{21} & \mathbf{Q}^*_{22} \end{pmatrix}$$

$$\Rightarrow \underline{\pi}_1^{\text{new}} := \underbrace{\underline{\pi}_1^{\text{old}}}_{\text{local memory}} \cdot \underbrace{\mathbf{Q}^*_{11}}_{\text{local disk}} + \underbrace{\underline{\pi}_2^{\text{old}}}_{\text{remote memory}} \cdot \underbrace{\mathbf{Q}^*_{21}}_{\text{local disk}}$$

- two threads interleave communication and computation
- barrier synchronisation after each iteration

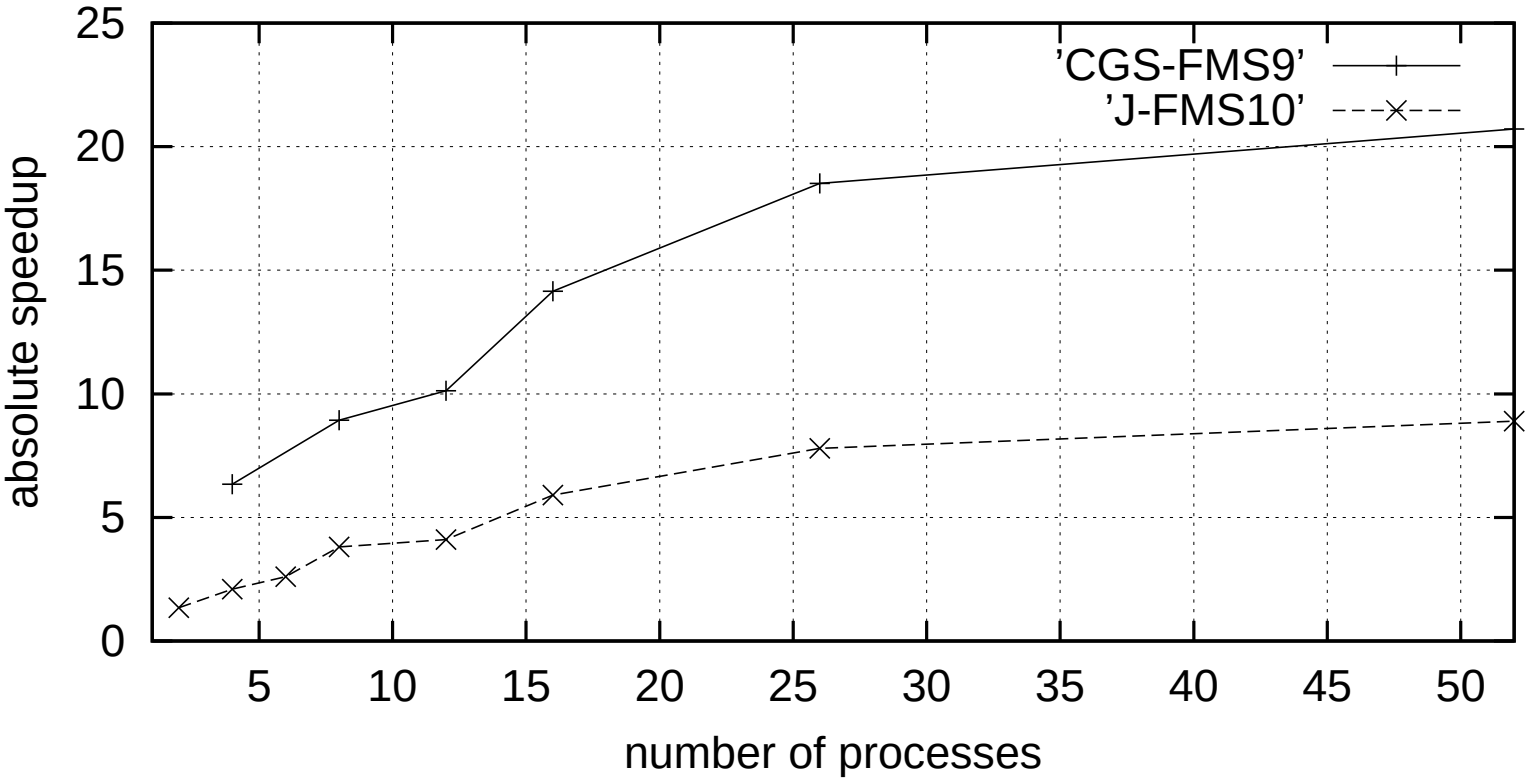
Distributed steady-state solution: I/O overhead

- FMS($k = 15$): $724\,284\,864$ states, $9\,134\,355\,680$ arcs
- solution method is really I/O-bound
- typical I/O-*overhead* per iteration:
  - 1200 MB read from local disk
  - 800 MB sent to other nodes
  - 800 MB received from other nodes
- 1 iteration $\approx$ 7 minutes; around 2000 required!
- for FMS($k = 10$): $25\,397\,658$ states: 7 seconds/iteration

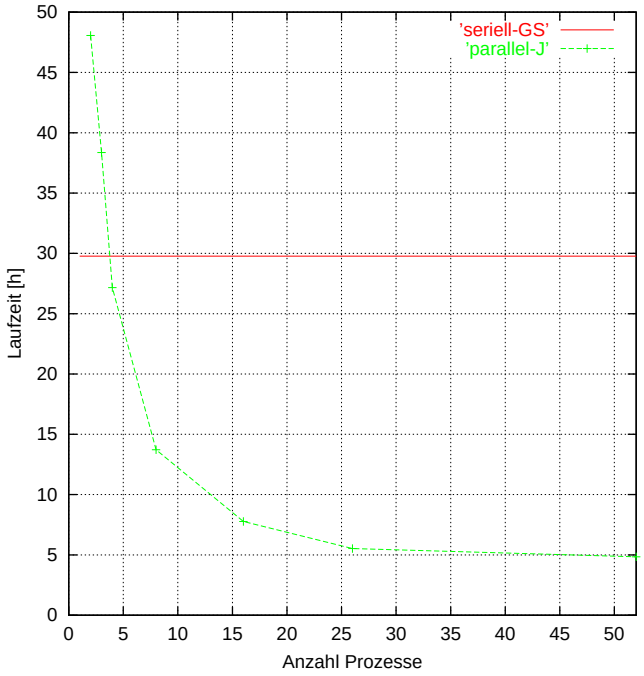
FMS distributed steady-state solution, $\epsilon = 10^{-8}$, $N = 26$

k	Jacobi				CGS			
	$\epsilon = 10^{-6}$		$\epsilon = 10^{-9}$		$\epsilon = 10^{-6}$		$\epsilon = 10^{-15}$	
	steps	time	steps	time	steps	time	steps	time
6	886	0:06:24	1507	0:10:53	371	0:05:22	893	0:12:55
7	1034	0:21:06	1783	0:36:23	150	0:07:09	1253	0:51:28
8	1181	0:43:56	1977	1:13:34	503	0:40:19	1469	1:57:44
9	1336	2:31:54	2208	4:11:03	863	2:52:59	1623	5:25:20
10	1492	7:36:36	2466	12:34:41	903	9:25:31	3559	37:08:53
11	1652	18:21:37	2722	30:15:08	665	15:04:44	3124	70:50:13
12	1818	47:59:51	2979	78:38:59	1219	65:00:17	2184	116:27:52
13	1990	68:53:55	3257	112:45:56	857	90:58:09	1987	210:55:02
14	2169	141:08:33	4448	289:26:34	325	72:36:53	—	—
15	2335	292:16:37	—	—	—	—	—	—

Distributed steady-state solution: speed-up



Kanban(7) distributed steady-state solution at $\epsilon = 10^{-7}$



[serial Gauss-Seidel: 624 iterations; distributed Jacobi: 1244 iterations]

Distributed steady-state solution: cross-arc impact

- Kanban model ($k = 7$): $41\,644\,800$ states, $450\,455\,040$ arcs
- with cross-arc percentage $X = 50\%$:

processors N	52	26	12	8
time/iteration	20	22	34	45

- with $N = 52$ processors:

cross-arcs [%]	98	94	70	51	30
time/iteration	34	28	23	20	14

Distributed steady-state solution: processor impact

- solution time and cross-arc percentage as function of the number of employed processors:

processors N	52	26	16	8	4	3	2
cross-arcs [%]	30	28.4	28.5	26.9	21.9	19	14.6
time/iteration	14	16	22.5	39.7	78.6	111	139.1

- serial solution with Gauss-Seidel: 171.5 seconds per iteration

- Jacobi requires more than 512 MB RAM; no serial solution available

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Summary

- completed first project on distributed evaluation of CTMCs from SPNs
- excellent performance for serial and distributed generation
- also excellent performance for distributed CTL model checking
- good performance for the distributed numerical solution, however, this remains the bottleneck
- main memory storage of solution vector(s) is the key issue
- further study of work-division strategy in solution speed

GRID-Based Challenges

- extension toward GRID-based model checking for CSRL:
  more complex algorithms and data structures (not just MVMs)
- three "levels" of access time: in node, in cluster, in grid
- complicated trade-off between communication (cross-arcs) and convergence speed
- combination with other techniques, e.g., for on-th-fly state-space reduction, bisimulations, etc.

Literature

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