Distributed On-the-Fly Resolution of Boolean Equation Systems

C. Joubert R. Mateescu

INRIA Rhône-Alpes / VASY
http://www.inrialpes.fr/vasy

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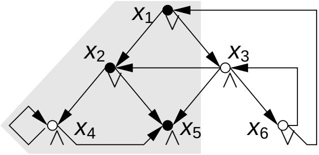
Outline

1. Boolean Equation Systems (BES)
   - Local Resolution of BES
   - Related Work
2. Distributed Local Resolution of BES
   - Distributed Algorithm DSOLVE
   - Implementation and Experiments

BESs, Boolean Graphs, and Local Sequential Resolution

$$\left\{\begin{array}{l} x_1 \stackrel{\mu}{=} x_2 \vee x_3 \\ x_2 \stackrel{\mu}{=} x_4 \vee x_5 \\ x_3 \stackrel{\mu}{=} x_2 \wedge x_5 \wedge x_6 \\ x_4 \stackrel{\mu}{=} x_4 \wedge x_5 \\ x_5 \stackrel{\mu}{=} \mathrm{T} \\ x_6 \stackrel{\mu}{=} x_3 \vee x_1 \end{array}\right.$$



- **BES** $\{x_i \stackrel{\sigma}{=} op_i X_i\}_{1 \leq i \leq n}$
  - Set of boolean fixed point equations
  - Pure disjunctive or conjunctive formulas
  - Absence of negations to ensure monotonicity of least fixed point
- **Boolean graph** $G = (V, E, L)$ **associated to a BES**
  - $V = \{x_1, \ldots, x_n\}$
  - $E = \{(x_i, x_j) | x_j \epsilon X_i\}$
  - $L : V \rightarrow \{\vee, \wedge\}, L(x_i) = op_i$

BES Application Areas

Verification of concurrent systems

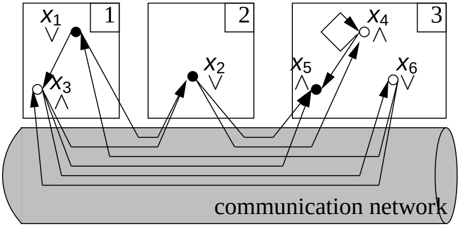
- Equivalence checking [Andersen-Vergauwen-95]
- Model-checking [Andersen-94]
- Partial order reduction [Pace-Lang-Mateescu-03]

Propositional logic programming

- Horn clauses satisfiability [Liu-Smolka-98]

Motivation for Distributed BES Resolution

1. **Limitation of Sequential BES Resolution Methods**
   - Memory
     - BES with more than 10^8 variables to solve (current sequential machines swap around 10^7 variables)
   - Time
     - Traversals of very large BES (the larger is the BES, the more resolution tasks will have the process)



2. **Reasons for Distribution**
   - Running faster with few memory used per machine
   - Regular problem prone to balanced distribution of tasks and data

Related Work

Distributed Local Resolution using Game Graphs
`[Bollig-Leucker-Weber-02]`

- Algorithm specialized for model checking properties expressed in a fragment of the modal μ-calculus
- Operates on game graphs ($\Rightarrow$ **Alternative representation to boolean graphs**)
  - Vertices = configurations in two-player games
  - Edges = moves
- Effective speedups obtained for model checking

The DSOLVE Algorithm

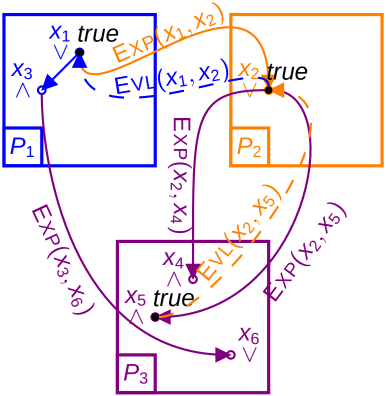
Computation model

- Distributed memory architecture (message passing): NOW, cluster of PC
- *P* SPMD processes and 1 coordinator process
- Each process solves a subset of boolean variables determined by a static hash function

Distributed algorithm

- Forward exploration of boolean graph (V, E, L) starting at a variable of interest $x \epsilon V$
- Backward propagation of stable (computed) variables
- Distribution (communication) of variables through remote dependencies
- Termination detection when x stable or boolean graph entirely solved

Execution of DSOLVE on previous BES example

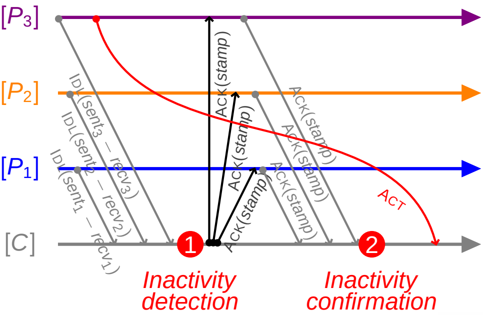


1. Initialization (variable of interest x_1)
2. Local expansion and remote expansion (message EXP)
3. Conjunctive variable whithout successor (i.e., constant **true**)
4. Local and remote (message EVL) back-propagation of stabilized (i.e., computed) variables
5. If variable of interest stabilizes, then resolution terminates

Distributed Termination Detection Algorithm

Principle

- Two waves of global inactivity detection between the coordinator and the resolution processes



Inactivity detection and confirmation

1. $\sum_{i=1}^{P} inactive_i = P \wedge \sum_{i=0}^{P} sent_i - recv_i = 0$
2. $nb(\text{ACK}(stamp)) = P \wedge stamp = current\_stamp$

Complexity Results

For a boolean graph (V, E, L) and P running processes:

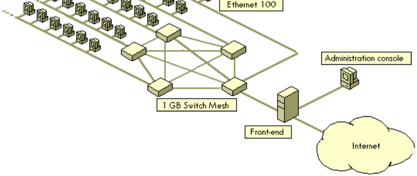
- Worst-case time complexity = $O(|V| + |E|)$
  - 2 intertwined graph traversals (forward and backward)
- Worst-case memory complexity = $O(|V| + |E|)$
  - Dependencies stored during graph exploration
- Worst-case message complexity = $O(2 \cdot |E| \cdot (P - 1)/P)$
  - 2 messages (expansion and stabilization) at most exchanged by edges
- Distributed termination detection = $O(|E| \cdot 3 \cdot P)$
  - 2 waves with $3 \cdot P$ messages at most exchanged by edge

Distributed Software Architecture

DSOLVE (10,000 lines of *C* code) is based on:

- Prototype generic communication library (4000 lines of *C* code) enabling communication through TCP/IP sockets
- Generic OPEN/CÆSAR environment for on-the-fly graph exploration `[Garavel-98]`, part of the CADP verification toolbox (`www.inrialpes.fr/vasy/cadp`)
- Generic boolean resolution API given by the library CÆSAR_SOLVE `[Mateescu-03]`
- Available under SOLARIS, LINUX, WINDOWS and MACOS operating systems

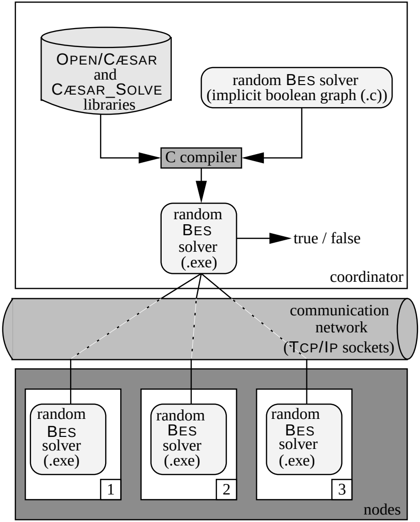
Platform Architecture



Experiments performed on homogeneous parallel architectures (NOW and clusters of PC), among which:

- IDPOT
  (`http://idpot.imag.fr`)
  48 Bi-Xeon 2.5 GHz 1.5 Gb
- ICLUSTER
  (`http://icluster.imag.fr`)
  216 PIII 733 MHz 256 Mb

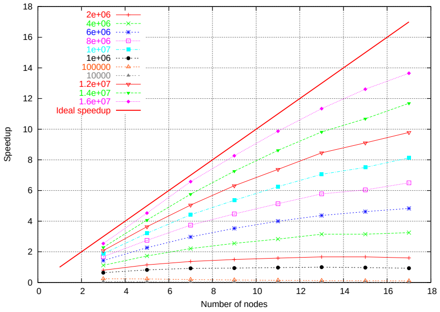
Experiments



Random BES Solver

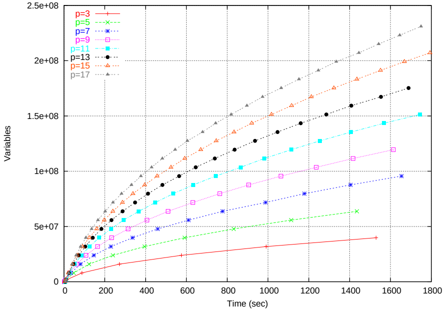
- Example of small application of DSOLVE (1000 lines of *C* code)
- Enables to test the performance of DSOLVE
- Produces BES (represented by the successor function of their corresponding boolean graph) according to various parameters which vary randomly in a given domain

Speedup in the Worst Case



- Worst case class of BES:
  - almost no alternation (2%), hence long path of ∨ (or ∧) variables ended by a constant T (or F) (1%)
  - encountered in model checking (CTL, ACTL, PDL)
  - encountered in equivalence checking involving deterministic graphs
- Quasi-linear speedup compared to the sequential breadth-first search algorithm of CÆSAR_SOLVE

Scalability



- Good scalability with the number of machines, as well as with the size of the BES to solve
- Experimented DSOLVE on more than 80 machines on ICLUSTER
- Solved $240 \cdot 10^6$ of variables and $1 \cdot 10^9$ operators in 28 minutes with 17 machines for a BES with 0% constant and 0% alternation

Memory and Communication Cost

- Low memory overhead of distributed resolution compared to memory allocated for data
- Perfect load balancing achieved by the static hash function
- High communication cost due to numerous cross-dependencies $((P-1)/P) \cdot |E|$
- Low percentage $(0,01\%)$ of termination detection messages over all exchanged messages

Summary

- DSOLVE: a new distributed local BES resolution algorithm
- Generic implementation within the CADP verification toolbox
- Linear worst-case time and memory complexity DSOLVE algorithm
- Extensive set of experiments showing linear speedups and good scalability

- Ongoing work
  - Generalizing DSOLVE to BES having several blocks of equations with acyclic inter-block dependencies
  - Developing other applications over DSOLVE, e.g. resolution of Horn clauses, model-checking and test case generation

For Further Reading

- D. Bergamini, N. Descoubes, C. Joubert and R. Mateescu.
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